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DOCTORAL THESIS



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Multiple stellar systems under photometric and astrometric analysis

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Chapter 1

Introduction

The binary stars are crucial for our knowledge about the universe. Especially eclipsing binaries provide us an unique insight to the basic physical parameters of the stars, stellar clusters, interstellar medium and galaxies. They are excellent distance indicators. We are able to learn more about the matter composition of the stars, about their evolution status, or the presence of planets or another components in these systems.

The very first task is the data acquisition, because only with the precise input data one is able to get the precise results. In last few decades mainly due to excellent satellite observatories (and not only in the visible part of the spectrum) our knowledge of them have rapidly grown.

On the other hand there is still decreasing the number of classical visual binary observations. Due to the new interferometers which could catch the milli- and micro- arcsecond angular distances, the semimajor axes of the astrometric binaries are still decreasing. Unfortunately, most of the systems analyzed below have the angular size of the astrometric orbit from 1 arcsec down to 100 mas, which is beyond the limits for the modern interferometers. And the lack of recent observations lead to the low accuracy of the results.

Another approach is the photometry and the classical observation of minimum light. Due to large "baseline" of observers in our country and the interest of amateur astronomers, the number of these observations is growing very rapidly and the cooperation between professional and amateur astronomers is very intensive. Also many observations of the minimum timings in this study came from amateur astronomers and these measurements are as accurate as from the professional observatories. Thanks to the large minimum times data set we are able to analyze many of the eclipsing binary systems for their long-term period variations.

The whole thesis is divided into several parts. In the first one is presented the theory needed for the analysis of multiple stellar systems by photometric and astrometric techniques, description of such systems and some limitations which have to be considered. In the second part are introduced several systems which show apparent period changes in their $O - C$ diagrams. And in the third one are the systems analyzed by the photometry and astrometry simultaneously. Also the catalogue of another suggested systems for the simultaneous analysis is included in this chapter. This is the crucial part of this thesis. The method itself is introduced in the chapter 2 and the results are in the chapter 4.

Chapter 2

Theory

2.1 Binaries

Most of the stars could be found as members of binary or multiple stellar systems. Recent analysis of large set of close binaries (Pribulla & Rucinski 2006) indicates that even most of the binaries are in multiple systems. According to this study about 59 % of northern-hemisphere contact binaries are members of the multiple stellar systems. The sample of binaries was analyzed very precisely, some distance-independent techniques were used, and the selection effects were also discussed.

During the last decades, many of the observational techniques become so effective and precise that it is possible to discover low-massive stars, brown dwarfs, or even exoplanets in eclipsing binary systems (hereafter EB).

The astrometric binaries are a special subset of binaries, where the individual components could be resolved into the separate stars, which means the angular separation of the components has to be above a certain limit of the technique used. Because the individual components in the system were discovered at different time epochs, they were marked by different labels. Most common is the use of A-B-C... sequence, which means that the component "A" was discovered as the first one, then was discovered the second one "B", and after then "C", etc. Sometimes one component was resolved to be a double, so it turns $A-B \rightarrow Aab-B$. Mostly A component is the brightest one. Here comes the problem with the hierarchy of such a system. Sometimes it is so-called *hierarchical-type* Aabc-B system (where B component is far away from the triple Aabc, where the c component could be far away from the ab double), and sometimes is so-called *trapezium-type* Aab-Bab system (two doubles Aab and Bab far away from each other), for the detailed description see e.g. Docobo & Andrade (2006).

In the present analysis another approach was used. It is based on the physical properties and gravitational bounding of the stars. The numbers 1 and 2 were used for the primary and secondary component of the eclipsing pair (these are not spatially resolvable) and index 3 as a label for the third, distant, component which is astrometrically observable. Sometimes also further component, the fourth, is suggested. In this study only the hierarchical systems are analyzed. It means that the higher the number of component, the bigger the semimajor axis (the component is away from the previous one)

$a_1 \ll a_2 \ll a_3 \ll \dots$. Because here we deal only with the relative astrometry (position angles and angular separation) it should be more precise to write $a_{12} \ll a_{12-3} \ll \dots$.

2.2 Principal methods for analyzing the EBs

Despite the fact that the introduced methods are the most important ones, only a short description was presented here. These methods are not the essential ones for the thesis. The two different methods (the astrometry and the analysis of times-of-minima observations) are the crucial and are described in detail in the next section.

2.2.1 Spectroscopy

Observing the spectra of the stars is still the most time-consuming activity performed on the astronomical observatories all around the world. Obtaining the spectrum of the star is also the most powerful tool for deriving the relevant parameters of the star. Thanks to different techniques of spectral analysis (spectrophotometry, line-profile analysis, disentangling, etc.) one can model the stellar atmosphere, derive the orbital parameters, or discover the surface structures. For the brief introduction to the topic and overview of the methods see e.g. Hilditch (2001).

Besides the spectral classification, the modelling of the radial velocity curves (hereafter RV) is one of the oldest methods of the spectral analysis. If only one component of the binary is observable, we deal with so-called SB1-type, and if both components are evident, it is called SB2-type binary. From both RV curves, we could derive many parameters of the relative orbits of the stars in the system (see Table 2.1 for the parameters and e.g. Wilson & Sofia (1976) for the method). Cross-correlation function method is now often used for analyzing the RVs, see e.g. McLean (1981).

Spectral disentangling (see e.g. Hadrava 1995) of composite spectra into the separate ones helps us to model the particular star in the system. Theoretical stellar models could be compared to the observed ones and one could study the physical conditions in the atmosphere of the star, temperatures, pressures, rotation, stellar wind or the chemical composition.

Another quite new technique is the Doppler profile mapping (see e.g. Rice et al. 1989). With this technique one is able to discover the dark and bright (cool and hot) spots on the surface of the star, as well as its rotation, or the evolution of the spots. Doppler tomography (see e.g. Marsh & Horne 1988) is able to detect similar structures and effects in accretion discs in interacting binaries. The evidence for discs, jets, or outflows could be observed in precisely measured spectrum of the star.

Table 2.1: The scheme of directly derivable entities.

	Only LC	Only		Only		LC		Only		Only $O - C$ LITE	Only $O - C$ Apsid.	Astrom.	LITE + Astrom.	LITE Apsid. Astrom.
		RV	SB1	RV	SB2	RV	SB1	RV	SB2					
$a_1 \sin i$ or $a_2 \sin i$		✓		✓	✓	✓	✓	✓	✓					
$a \sin i, a_1 \sin i, a_2 \sin i, M_1 \sin^3 i, M_2 \sin^3 i$				✓	✓									
$a, a_1, a_2, M_1, M_2, R_1, R_2, \mathcal{L}_1, \mathcal{L}_2, d$						(✓)								
$P, e, \omega, (\dot{\omega})$	✓	✓		✓	✓	✓	✓	✓	✓		✓			✓
γ		✓		✓	✓	✓	✓	✓	✓					
q	(✓)			✓	✓	(✓)								
$i, \frac{R_1}{a}, \frac{R_2}{a}, L_1/L_2, g_1, g_2, A_1, A_2, F_1, F_2, x_1, x_2, l_3$	✓					✓	✓	✓	✓					
T_2	✓	?		?	?	✓	✓	✓	✓					
JD_0, P	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓		✓	✓
$p_3, T_0, A_3, \omega_3, e_3, f(M_3), M_{3,min}$										✓			✓	✓
$p_3, T_0, \omega_3, e_3, i_3, \Omega_3, a_3''$												✓	✓	✓
M_3, D													✓	✓

Some comments: The table shows which entities could be derived when one has the measurements of the particular type. The table is divided into two parts. In the first one (the left top corner) are the parameters and methods adopted from Kallrath & Milone (1999), page 142. In the second part are the parameters of the third-body orbit which could be derived from the methods used in this thesis. 'LC' stands for the light curve, 'RV' for the radial velocity curve, 'SB1' and 'SB2' for the types of the spectroscopic binaries, 'LITE' for the light-time effect, 'Apsid.' for the apsidal motion and 'Astrom.' for the astrometric orbit, respectively. In the parameters column a_1 and a_2 denotes for the semimajor axis of the primary and secondary component in the EB, i for the inclination of the EB orbit, M_1 and M_2 masses of the primary and secondary, R_1 and R_2 radii of components, \mathcal{L}_1 and \mathcal{L}_2 for the bolometric luminosities, d for the separation between the components, e for the eccentricity of the EB orbit, ω for the argument of periastron of the orbit, γ is the systemic velocity of the EB pair, q is the photometric or the spectroscopic mass ratio, L_1 and L_2 monochromatic luminosities in a specified passband, g_1 and g_2 gravity darkening coefficients, A_1 and A_2 albedo coefficients, F_1 and F_2 rotation parameters, $\bar{F}_i = \frac{\omega_i}{\omega}$, x_1 and x_2 limb-darkening coefficients, l_3 the third light and T_2 temperature of the secondary (T_1 fixed), respectively. The only one parameter was added compared with Kallrath & Milone, the apsidal motion rate $\dot{\omega} = \frac{d\omega}{dt}$, and the parentheses mean that the parameter is computable only if some apsidal motion is presented. Sometimes it is difficult to compute the parameters, or the ability to compute them depends on the type of the EB. For example the photometric mass ratio could be computed only when the EB is contact or over-contact type (that is the reason why there are the parenthesis). The question mark indicates that the temperatures could be derived from the spectrum of the star. The parameters from the second part are explained in the text, D stands for the distance to the system.

Sometimes also an additional component is observable in the spectra. The spectral lines of the third component are staying on the same wavelength, while the lines of the binary components are moving in agreement with the actual orbital phase of the EB. In the more favourable case also the third lines move very slowly according to the third-body orbit. In some others only the barycenter of the whole system is moving very slowly in agreement with the phase of the third-body orbit (see Eq. 2.7 in chapter 2.3.4). Such effects are hardly observable and their final detection is questionable in most of the cases (see Mayer (2004) for the list of such systems and the subsections 4.2 and 4.4 below).

2.2.2 Photometry

Modern photoelectric and CCD photometry is a very powerful tool to study the eclipsing binaries. Thanks to very wide network of amateur astronomers with their CCD cameras all around the world, it is nowadays quite easy to observe the whole light curve of the particular eclipsing binary in the standard filters and perform a detailed analysis of the system.

From the light curve (hereafter LC) one could derive many useful parameters of the eclipsing binary itself. The comparison between the parameters derivable from the LC solution, from the RV fitting and from the techniques described below is shown in Table 2.1.

From this set of parameters, together with the radial velocity curves, one could obtain a complete set of parameters describing the orbit of the individual components in the EB, as well as the basic physical parameters of both components in absolute units. The masses, the radii, the luminosities and the temperatures could be calculated, if the precise photometry together with both radial velocities were carried out (see Table 2.1). The limb-darkening coefficients (see e.g. van Hamme 1993), as well as gravity brightening (see e.g. Lucy 1967) and albedos (see e.g. Ruciński 1969) of the components could be also calculated from the model. The most common codes for the EB modelling are the Wilson-Devinney (Wilson & Devinney 1971), FOTEL (Hadrava 2004), Linnell's model (Linnell 1984), LIGHT (Hill 1979), etc.

Modern advanced tools for analyzing the light curves of EBs are very powerful for discovering the surface structures and their evolution. Dark or hot spots on the star surface could be implemented into the model (see e.g. Poe & Eaton 1985).

The precise photometry of the binary in different filters could also reveal the third light from the distant component in the LC solution. The third light l_3 could be the indicator or the proof for the presence of the third body suggested by some independent method.

2.2.3 Additional techniques

Besides the photometry and spectroscopy also some another methods play a role in our modern knowledge about the EBs. The invisible part of the spectrum was also used to study these objects. Some of the EBs were successfully identified to be radio- or X-ray active. Such systems are mostly the complicated ones, currently in the unstable

evolutionary status, under rapid mass transfer, having an accretion disc or be a member of a cataclysmic variable.

Another approach is the use of polarimetry (see e.g. Hall 1949), or magnetometry (see e.g. Marcy 1984) to study EBs. These two modern techniques are also very powerful and could reveal some properties of these systems which are otherwise undetectable, for example the disc or stream in the system, large coronae of one of the components, or magnetic fields.

2.3 $O - C$ diagram analysis

The periodic behavior of the eclipsing binaries could be studied. The long-term changes of the binary period could be caused by various effects and these effects could be analyzed due to the large database of times-of-minima observations which cover more than a century in many cases. Especially due to amateur astronomers with their CCD cameras in the recent few years the number of times of minima is growing rapidly, because these observations are much easier to obtain than the photometric observation of the whole LC. The accuracy of such observations are sometimes not very good (mainly the old ones), so the analysis is also problematic. The topic was discussed in detail for example in Sterken (2005).

The $O - C$ diagram in our case is the special plot, where in the x-axis is mostly the epoch (or the date or Julian date), while in the y-axis is the difference **O**bserved minus **C**alculated. Here in the case of the eclipsing binaries this means the difference in times of minima in the particular system, expected minus predicted moment of minimum light.

The linear ephemeris of the binary indicates the minimum time JD after E cycles (E is the epoch number) since the initial time of minimum JD_0 occurred. One could write

$$JD = JD_0 + P \cdot E,$$

where P is the period of the eclipsing binary. The value $O - C$ is therefore defined as a difference

$$O - C = JD - JD_0 - P \cdot E.$$

Finding new revised linear ephemeris of the binary could be also done with these equations. One has to minimize the sum

$$\sum_{i=1}^N (O - C)_i^2 \rightarrow 0$$

with respect to the parameters (JD_0 , P) over the whole parameter space and N is the number of times of minima used.

2.3.1 Constant period

One could expect the constant period and therefore the linear trend in the $O - C$ diagrams in most of the eclipsing binaries. If the assumed period of the EB is lower than the right one, the linear trend in the diagram is increasing, while the proposed period is higher

than the right one, the times of minima are decreasing in the diagram. The collection of many $O - C$ diagrams of the EBs is for example in Kreiner's Atlas of $O - C$ diagrams, see Kreiner et al. (2001).

Sometimes the $O - C$ diagrams show the changes of periods which seems to be abrupt. Such "jumps" in the diagram could be caused by the sudden period changes. These are often explained as a mass ejection from one star to another. Sometimes this explanation is used only due to poor coverage of the abrupt change in the diagram, sometimes is confirmed by another independent method.

2.3.2 Mass transfer

Another phenomena which could be studied in the $O - C$ diagram is the parabolic behavior of times of minima. If the data set is large enough, in some binaries it is possible to identify the parabolic (increasing or decreasing) trend, which means that the period of the binary is steady increasing or decreasing. In $O - C$ diagram it means

$$JD = JD_0 + P \cdot E + q \cdot E^2,$$

where q is the quadratic term coefficient. It is possible to show that this additional term could be caused by the mass transfer in the binary.

There are two basic kinds of mass transfer between the components. The first (and the simplest) one is the *conservative mass transfer*. In this case the total mass of the binary as well as its total orbital angular momentum are conserved in the system. The other one is the *nonconservative mass transfer*, where these quantities are not constant for the whole system. A few mechanisms could cause this latter case: stellar wind, Roche-lobe overflow, or a sudden catastrophic event. This kind of mass transfer is probably more often in the nature.

Concerning the former case, from the quadratic term coefficient q also the conservative mass transfer rate could be derived

$$\dot{M}_1 = \frac{M_1 M_2}{3(M_1 - M_2)} \cdot \frac{\dot{P}}{P} = \frac{2q M_1 M_2}{3P^2(M_1 - M_2)}.$$

The brief introduction to the topic and derivation of the equations for both cases could be found in Hilditch (2001), page 162. The typical values of mass transfer rate are circa $10^{-7} - 10^{-9} \text{ M}_\odot/\text{yr}$.

2.3.3 Apsidal motion

The next effect which could be studied only on the basis of the $O - C$ diagram analysis is the apsidal motion. In some eccentric binaries the line of apsides of the orbit of such a system is moving in space. One has to take into consideration the two different periods, the sidereal one P_s and the anomalistic one P_a , which are in relation

$$P_s = P_a(1 - \dot{\omega}/2\pi).$$

The quantity $\dot{\omega}$ is the apsidal motion rate.

Such an effect is easily detectable in the $O-C$ diagram of the binary. Both primary and secondary minima are being periodically shifted from the linear ephemeris and also against the other one (the primaries and secondaries are in anti-correlation). The necessary equations for the apsidal $O-C$ diagram analysis are presented in Gimenez & Garcia-Pelayo (1983).

2.3.4 Light - time effect

Large set of times of minima of the EBs could be also used for discovering the additional component(s) in these systems. With the light-time effect (hereafter LITE) analysis one can suppose a presence of another component(s) in the system only by analyzing the times of minima and their long-time behaviour. The motion of EB around the common barycenter causes apparent changes of the observed binary period with a period corresponding to the orbital period of the third body.

Irwin (1959) improved a method by Woltjer (1922) for analyzing the long-term variation of the times of minima caused by the third body orbiting the eclipsing pair. Very useful comments and limitations were discussed by Frieboes-Conde & Herczeg (1973) and by Mayer (1990). Nowadays there are several dozens of EBs, where the LITE is certainly presented or supposed (see e.g. Borkovits & Hegedues 1996, Albayrak et al. 1999, Wolf et al. 2004).

From the numerical point of view the method is a classical inverse problem. We have M measurements of the times of minima of the system at certain constant JD_i with the individual uncertainties $\sigma_{m,i}$. Our task is to find five parameters describing the orbit of the third body in the system: the period of the third body p_3 , the LITE semiamplitude A , the eccentricity e , the time of the periastron passage T_0 , and the longitude of periastron ω_{12} for the binary on its orbit around the common barycenter. We have to compute simultaneously also two (or three) parameters of the eclipsing binary itself, namely its linear (or quadratic) ephemeris JD_0 and period P (and q for the quadratic one). Altogether, one has 7 (or 8) parameters to derive from the model fit of the minimum-time measurements

$$\{(JD_i, \sigma_{m,i})\}_{i=1,M} \rightarrow (p_3, A, e, T_0, \omega_{12}, JD_0, P, q). \quad (2.1)$$

The least-squares method and the simplex algorithm (see e.g. Kallrath & Linnell 1987) were used. The basic mathematic equations are the following. Compute the mean anomaly from the time of the measurement (the subscript i was omitted for the sake of brevity)

$$M = 2\pi \cdot \frac{(JD - T_0)}{p_3}. \quad (2.2)$$

Then solve the Kepler equation $M \rightarrow E$ and after then convert the eccentric anomaly to the true anomaly of the third body in its orbit

$$\nu = 2 \cdot \arctan \left(\sqrt{\frac{1+e}{1-e}} \cdot \tan \frac{E}{2} \right). \quad (2.3)$$

After then, one can use the formula

$$\Delta\tau = \frac{A}{\sqrt{1 - e^2 \cos^2 \omega_{12}}} \cdot \left[\frac{1 - e^2}{1 + e \cos \nu} \sin(\nu + \omega_{12}) + e \sin \omega_{12} \right] \quad (2.4)$$

to compute the magnitude of the LITE. Now it is possible to calculate the difference between the observed and calculated time of minimum

$$(O - C) = JD - JD_0 - P \cdot E - q \cdot E^2 - \Delta\tau, \quad (2.5)$$

where E is the epoch of the JD according to the ephemeris JD_0 and P , and q is the quadratic term quotient. The resultant sum of normalized square residuals is

$$\chi_{LITE}^2 = \sum_{i=1}^M \left(\frac{(O - C)_i}{\sigma_{m,i}} \right)^2. \quad (2.6)$$

Our task is to minimize this value and find the set of parameters $(p_3, A, e, T_0, \omega_{12}, JD_0, P, q)$ describing the orbit.

The weighting is provided by the uncertainties $\sigma_{m,i}$. These values are obtained from the observations, or estimated as some typical uncertainty level for the certain kind of measurement provided by specific instrument. Another way is the following method. If we have information about the type of the observation (the method how the measurement was obtained), we could use some weighting scheme w_i instead of uncertainties (e.g. $w_i = 1$ for visual and $w_i = 10$ for photoelectric/CCD measurements) and solve the corresponding problem. From this solution we could find the uncertainties $\sigma_{m,i}$ simply as a differences between the observed and the predicted values.

The third body in the system also causes variations in gamma velocities. If we know v_γ from different RV investigations and in different epochs of the system, we could see a variation of v_γ with a period corresponding to the orbital period of the third body. The variation could be described by

$$v_\gamma = K[\cos(\nu + \omega_{12}) + e \cos \omega_{12}], \quad (2.7)$$

where $K[\text{km}\cdot\text{s}^{-1}]$ is the amplitude of such variation and could be calculated from the LITE parameters $A[\text{d}]$, $p_3[\text{yr}]$, e and ω_{12} from the equation

$$K = \frac{A}{p_3} \cdot \frac{5156}{\sqrt{(1 - e^2)(1 - e^2 \cos^2 \omega_{12})}}. \quad (2.8)$$

But the basic limitation is very often the long period p_3 , which is usually too long to have reliable RV data for this analysis.

LITE hypothesis could be also applied to another components in the system. The third body may cause LITE₃ and another (fourth) component cause LITE₄. The resultant total effect is then simply the sum of the two effects $\text{LITE} = \text{LITE}_3 + \text{LITE}_4$. One necessary condition has to be satisfied. The fourth component has to be more distant than the third one (and the third one distant from the EB pair). This is the main physical condition,

because the method itself was derived with this condition by the use of von Zeipel's method to the three-body problem.

If one adopts another effects, which could play a role in the problem, the individual terms could be easily added. This means if one wants to describe a system with the LITE caused by the third and the fourth component, also an apsidal motion is presented and mass transfer together, the variation in $O - C$ diagram could be then described as

$$(O - C) = JD - JD_0 - P \cdot E - q \cdot E^2 - (O - C)_{LITE,3} - (O - C)_{LITE,4} - (O - C)_{apsid}.$$

One has to distinguish between primary and secondary minima in apsidal motion term, which could be computed according to equations from Gimenez & Garcia-Pelayo (1983).

2.4 Astrometry

Another method to study binaries and the properties of their orbits is astrometry. The number of visual binaries with astrometric orbits has grown, but the whole orbit is covered with data points only in some cases. Thanks to precise interferometry the observable semimajor axes of astrometric binaries are still decreasing down to milli- and micro- arcseconds. On the other hand, one has to regret that no recent astrometric measurement of a wide pair of about $1''$ has been obtained for the systems mentioned below. Most of the astrometric observations were adopted from The Washington Double Star Catalogue WDS, see <http://ad.usno.navy.mil/wds/> (Mason et al. 2001). The first astrometric observations of visual doubles are a few centuries old. Altogether there are about 2000 systems with their visual orbits known.

The astrometric measurements of binaries used consist of a series of measurements of position angle θ_i and separation ρ_i secured at different times t_i ($i = 1, N$). Sometimes also the errors of the individual data points are available. The weighting scheme is provided by using the uncertainties σ_θ and σ_ρ of the individual observations.

From the astrometric data one is trying to find the parameters of the relative orbit, defined by 7 parameters: period p_3 , angular semimajor axis a , inclination i , eccentricity e , longitude of the periastron ω_3 for the third body on its orbit, the longitude of the ascending node Ω , and the time of the periastron passage T_0 . One has to solve the inverse problem

$$\{(t_i, \theta_i, \rho_i, \sigma_{\theta,i}, \sigma_{\rho,i})\}_{i=1,N} \rightarrow (a, p_3, i, e, \omega_3, \Omega, T_0). \quad (2.9)$$

The least-squares method and the simplex algorithm was used. The basic mathematic equations are the following. Compute the mean anomaly from the time of the measurement, according to Eq. 2.2, solve the Kepler equation $M \rightarrow E$ and after then convert the eccentric anomaly to the true anomaly, according to Eq. 2.3. Compute the radius vector in arcseconds (the subscript i was omitted for the clarity)

$$r = a \cdot \frac{1 - e^2}{1 + e \cos \nu}, \quad (2.10)$$

and from this equation one can compute the position on the sky, θ and ρ , respectively:

$$\tan(\theta - \Omega) = \tan(\nu + \omega) \cdot \cos i \quad (2.11)$$

$$\rho = r \cdot \cos(\nu + \omega) \sec(\theta - \Omega). \quad (2.12)$$

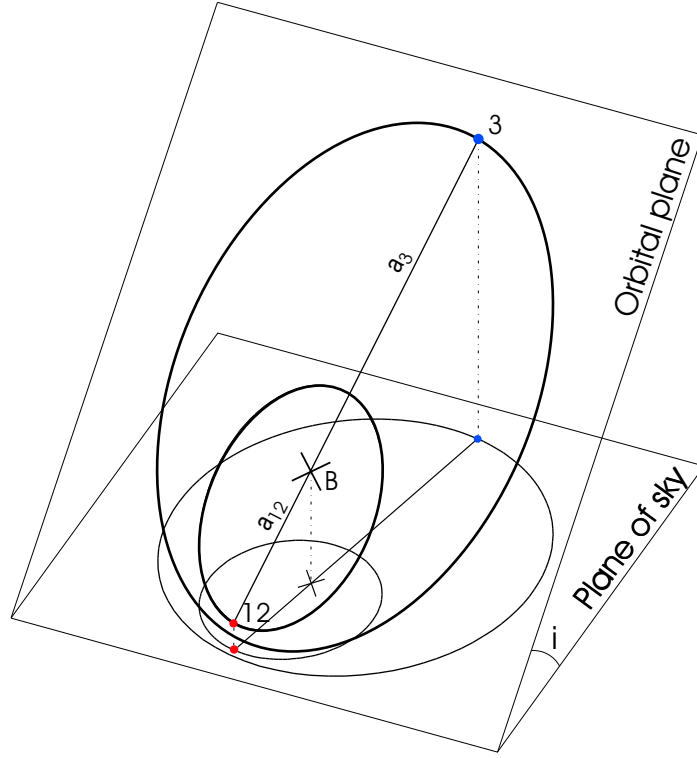


Figure 2.1: A simplified description of the relative binary orbit on a plane of the sky. The true orbit is inclined against the plane of the sky (angle i). B denotes for barycenter of the whole system, 12 denotes for the eclipsing binary pair and 3 for the third component. The picture shows the moment, when the bodies are in the apocenters on their respective orbits. Also the semimajor axes of the third body and EB on the long orbit (a_{12} and a_3) are shown.

Comparing this theoretical position on the sky with the observed ones θ_0 and ρ_0 , one can calculate the sum of normalized residuals squared

$$\chi_{astr}^2 = \sum_{i=1}^N \left[\left(\frac{\theta_i - \theta_{0,i}}{\sigma_{\theta,i}} \right)^2 + \left(\frac{\rho_i - \rho_{0,i}}{\sigma_{\rho,i}} \right)^2 \right], \quad (2.13)$$

following Torres (2004). With this χ_{astr}^2 and using the simplex algorithm (see e.g. Kallrath & Linnell 1987) one can obtain a set of parameters $(a, p_3, i, e, \omega_3, \Omega, T_0)$ describing the astrometric orbit.

At this place it is necessary to remark one useful comment. One has to distinguish between the two angles ω_3 and ω_{12} . The parameter used in LITE analysis is ω_{12} , but in astrometry the quantity $\omega_3 = \omega_{12} + \pi$ is employed. In this thesis the angle ω stands for the longitude of the periastron for the eclipsing binary, i.e. $\omega = \omega_{12}$, and the subscripts will be omitted for clarity.

2.5 Combining the methods

Our task is to combine the astrometry and the analysis of times of minima into one joint solution together. If one has N astrometric and M minimum-time measurements, it is possible to merge them together and obtain a common set of parameters

$$(t_i, \theta_i, \rho_i, \sigma_{\theta,i}, \sigma_{\rho,i}, JD_i, \sigma_{m,i}) \rightarrow (A, p_3, i, e, \omega, \Omega, T_0, JD_0, P, q). \quad (2.14)$$

This set of 10 parameters fully describes the orbit of the eclipsing binary around the common center of mass with the third unresolved component together with the ephemeris of the binary itself.

One is also able to find out the mass of the third body and the semimajor axis of the wide system because the inclination is known and one can calculate the mass function of the wide orbit

$$f(M_3) = \frac{(a_{12} \sin i)^3}{p_3^2} = \frac{(M_3 \sin i)^3}{(M_1 + M_2 + M_3)^2} = \frac{1}{p_3^2} \cdot \left[\frac{173.15 \cdot A}{\sqrt{1 - e^2 \cos^2 \omega}} \right]^3, \quad (2.15)$$

where a_{12} stands for the semimajor axis of the binary orbit around the common center of mass and M_1, M_2, M_3 are the masses of the primary, secondary, and tertiary component, respectively. For more details see e.g. Mayer (1990).

The only difficulty which remains unclear is the connection between the angular semimajor axis a and the LITE amplitude A . The quantity a_{12} could be derived from Eq. 2.15 and with the masses of the individual components one is able to calculate also the value a_3 , i.e. the semimajor axis of the third component around the barycenter of the system

$$a_3 = a_{12} \cdot \frac{M_1 + M_2}{M_3}. \quad (2.16)$$

The total mutual distance of the components is $a_{total} = a_{12} + a_3$ (see Fig.2.1). Using Hipparcos parallax π (Perryman & ESA 1997) one can obtain the distance D to the system. Now it is possible to enumerate the angular semimajor axis a as a function of D and a_{total}

$$a = \arcsin \left(\frac{a_{total}}{D} \right). \quad (2.17)$$

The way how the two different approaches were put together is following the similar approach by Torres (2004). From the mathematical point of view both methods are analogous and there is an overlap of the parameters in both methods. Our task is to minimize the combined χ^2

$$\chi_{comb}^2 = \chi_{astr}^2 + \chi_{LITE}^2, \quad (2.18)$$

where χ_{astr}^2 and χ_{LITE}^2 are the sums of squares according to Eqs.2.13 and 2.6.

Sometimes there arises a problem with the χ^2 values in both methods, which are incomparable. Especially when there are much more data points in one method against the other, also the resultant χ^2 would be much larger and as a consequence this method outweigh the other one. This problem could be eliminated using new uncertainties σ instead of the old ones

$$\sigma_{\theta,i} \rightarrow \sqrt{N} \cdot \sigma_{\theta,i}, \quad \sigma_{\rho,i} \rightarrow \sqrt{N} \cdot \sigma_{\rho,i}, \quad \sigma_{m,i} \rightarrow \sqrt{M} \cdot \sigma_{m,i}.$$

2.5.1 Error estimation

The errors of the output parameters were calculated according to the method described in Numerical Recipes, pages 684 - 694, see Press et al. (1986). Using the confidence limit 95%, one could calculate

$$1 - 0.95 = \Gamma(\nu/2, \Delta\chi^2/2),$$

where Γ is the incomplete gamma function, ν is the number of parameters fitted and $\Delta\chi^2$ defines the boundary around the final solution. The area within this boundary is scanned and the maximum value of difference between actual parameter and final parameter $\delta\alpha = (\alpha_i - \alpha_0)$ was taken as an error of the particular parameter.

2.6 Distance determination

Another task is determining the distance to these systems, which could be also done by combining the astrometry and the LITE analysis. As was mentioned above, from the LITE analysis it is possible to derive the quantities $a_{12} \sin i$ and also $M_3 \sin i$, and from the astrometric analysis it is possible to compute the angle i and determine the semimajor axis and the third mass in absolute units. Thanks to these values the total semimajor axis a_{total} (see above) could be determined. Comparing the value a_{total} with the angular semimajor axis of the binary on the sky and leaving the parallax of the system as another free parameter, one could compute this value and derive the distance to the system.

But this method is useful only in very special cases, where at least one period is covered with data points sufficiently in both methods and both methods give us precise and comparable results. If the methods produce different results, the method is not useful for the distance determination.

2.7 Limitations of the methods

The methods themselves are very powerful and useful, but one has to take into consideration also some physical and observational limitations.

If one considers only the LITE, the main observational limitation are the amplitude A and period p_3 . There are a few often-used methods to determine the time of minimum, the most common is Kwee-van Woerden method (Kwee & van Woerden 1956). Its main advantage is that one can compare the results and the individual errors of the measurements, or recalculate the old data. It is suitable only for symmetric minima.

For a different types of eclipsing binaries we could reach a different accuracy of the times of minima. Using the precise photoelectric and CCD detectors one is able to compute the time of minimum light with precision of about 1–10 seconds or less (≈ 0.0001 – 0.00001 day). This is only the theoretical value, if there are no clouds, or the moon, and the observational conditions are very good.

The practice is sometimes quite different. If one compares two observations of the same star and the same minimum time (after the transformation to the heliocentric Julian time), one finds out that there could be a difference between them in order 0.005 day.

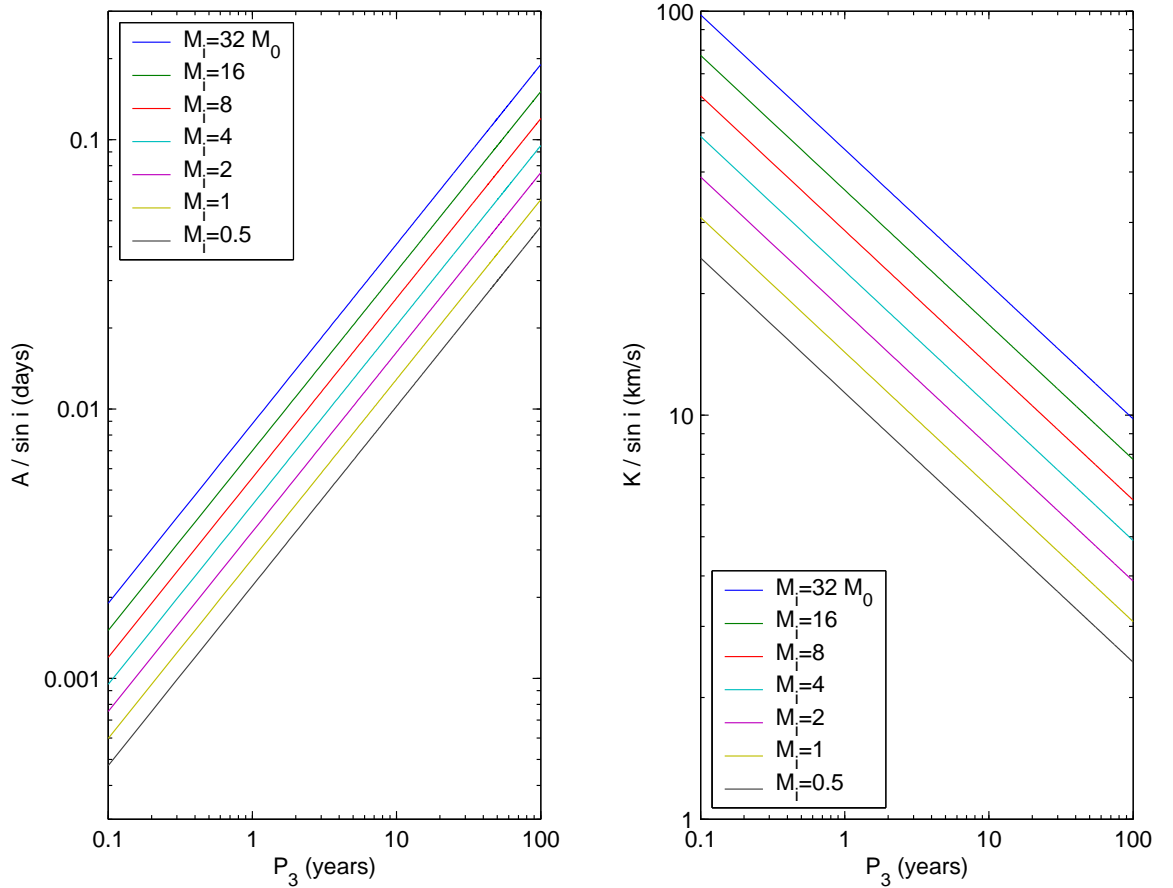


Figure 2.2: In the left figure are the physical limits of amplitude of LITE as a function of period and mass. The different lines represent different masses of the components (for $M_1 = M_2 = M_3$). In the right one the same for amplitude of systemic velocity variations. The figure adopted from Mayer (1990).

So there is a question about the accuracy of the method and the true error. Especially amateur astronomers have sometimes very good measurements, but their results (the times of minima) are not very satisfactory. This could be due to shift of time on their computer (the delay could be from a few to tens of seconds!), or the wrong method used for determining the time of minimum. Another possible explanation is, that the time of exposure could be the beginning of the exposure (instead of the middle of exposure).

Especially due to this reason there is a principal limit of the amplitude of LITE which could be reached. The amplitudes of LITE below 0.01 day are problematic, but in some systems also lower amplitudes are detectable (e.g. RT And or RZ Com). It is also necessary to take into consideration that the old measurements are not photoelectric, but visual and photographic, where the errors of the individual data points are much larger (and these errors are mostly not available for the analysis).

Considering only LITE, the plots with the relative limitations are in Fig. 2.2. One can see the dependence of the amplitude and the period on the masses of the individual components (only the case with equal masses is shown).

If one uses the combined approach, it is necessary to discuss also the astrometric orbit

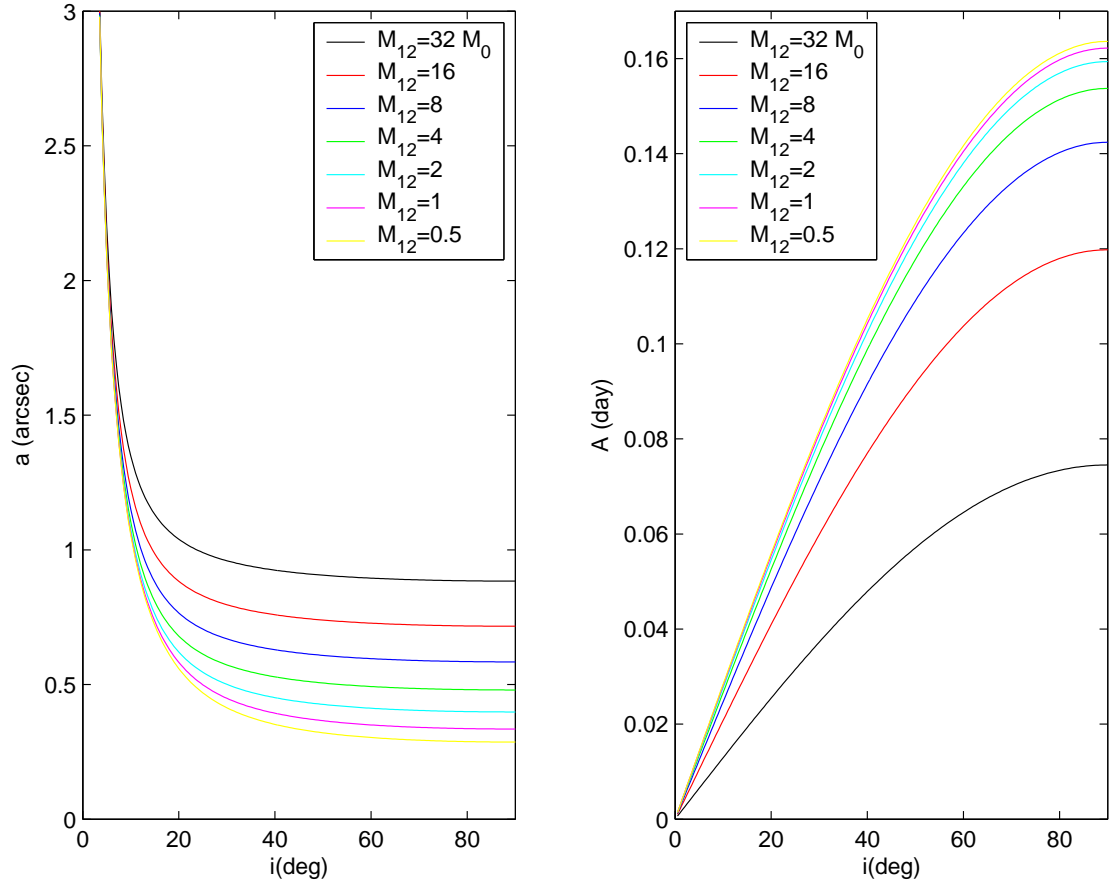


Figure 2.3: The principal limits of amplitude of astrometric orbit and amplitude of LITE in combined solution. The left figure was plotted with fixed LITE amplitude $A = 0.03$ d and the period $p_3 = 20$ yr. The right one with the same period and the amplitude of astrometric orbit $a = 1$ arcsec. The value of parallax was fixed, $\pi = 35$ mas. The different curves represent the different masses of the EB, where the introduced masses are $M_{12} = M_1 + M_2$.

and the accuracy of the astrometric measurements. New interferometric techniques are very precise and could reach a few miliarcseconds (mas), but the older data are visual or photographic. In most cases the errors of individual data points are unavailable.

Figure 2.3 shows the combined solution and the amplitudes in both methods. They are strongly dependent on the inclination i of the orbits. In the first one the LITE amplitude was fixed, while in the second one the astrometric-orbit amplitude was fixed. The basic properties of the plots in the diagrams are due to the geometry of the system. The edge-on orbit ($i \rightarrow 90^\circ$) leads to the binary eclipses, while the face-on orbit ($i \rightarrow 0^\circ$) leads to the most pronounced astrometric variation. The parallax (the distance) of the system was also fixed. One has to consider also the limitations of the methods described above and the areas in the diagrams where these limits are satisfied.

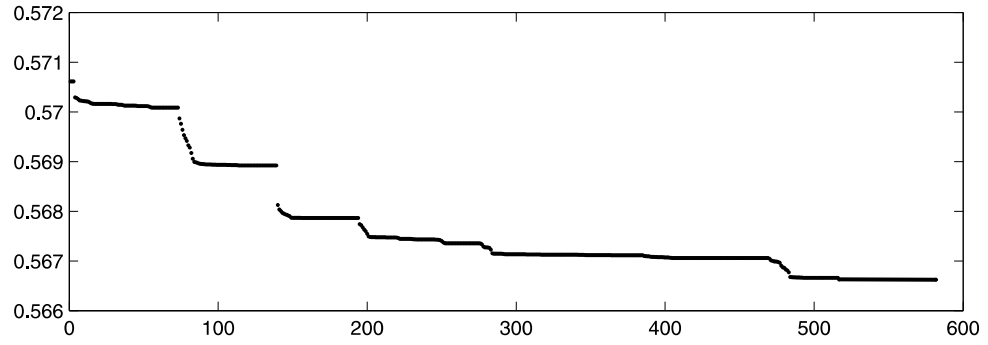


Figure 2.4: The sum of squares of residuals as a function of number of iterations, the case VW Cep *Solution I.* (see below).

2.8 Numerics and the strategy to solve the problem

The efficiency of the combined method and the computing time required by the algorithm strongly depend on the initial set of parameters and the input data. If nothing is known about the solution, one has to scan a wide range of parameters (eccentricity e from 0 to 1, and the angular parameters from 0° to 360° , etc.).

The efficiency of the algorithm could be improved if the simplex is used repeatedly. It can happen that the simplex converges into a local minimum while the global one is far away. It is therefore advisable to run the algorithm again, with as large initial steps as in the previous run, but keeping the values of the parameters corresponding to the previously found minimum as one vertex. Repeating this strategy several times over the whole parameter space, one can judge whether the global minimum was found by checking whether the sum of squares of the residuals is still changing or not (see Fig. 2.4 and 2.5).

If one could guess the approximate values of the parameters, it is also recommended to use them. If one do not have any information about their values, the algorithm itself is able to find the appropriate ones, but these could be only the numerical ones without any physical meaning. It is recommended to set the initial values of parameters as close as possible to the right values, because the algorithm itself will converge faster and the probability of the code to produce incorrect parameters is lower.

Another problem is the number of parameters used. The number of parameters strongly affects the computing time required. Regarding the classical LITE problem, one has 7 parameters (2 from ephemeris and 5 from LITE), but using the quadratic term also one further parameter is necessary to evaluate. One can estimate the ephemeris parameters (JD_0, P, q) in the first step and after then with fixed ephemeris calculate the parameters of LITE itself. But it is strongly recommended to compute all parameters together, because the LITE could also affect the ephemeris and change the values slightly. On the other hand sometimes it is better to fix the values of some parameters for the code to be faster. However, it is necessary to release all of the parameters for the final fit.

If also another effect appears in the analysis, for example apsidal motion, it is necessary to estimate another 3 parameters ($\omega_0, e', \dot{\omega}$). For the combined analysis, there could be

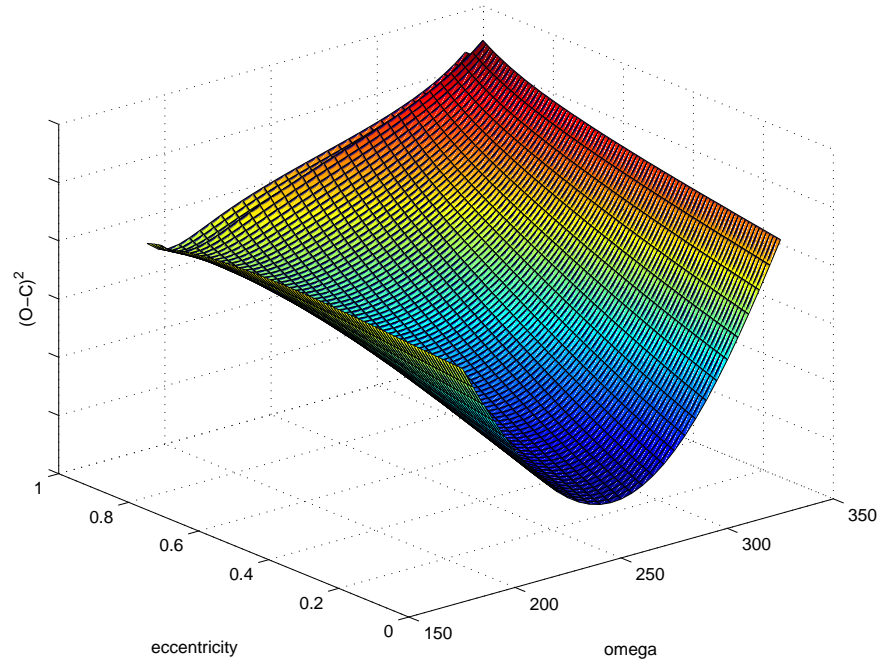


Figure 2.5: The sum of squares of residuals as a function of ω and e for the third-body orbit.

even more parameters. In the most complicated case, for VW Cep see section 4.2, there were 14 parameters which have to be determined. The strategy which helps the code to converge faster to the minimum could be shown in the case HT Vir (see below section 4.5). There is the astrometric orbit known with high precision, so the astrometric parameters ($a, p_3, i, e, \omega, \Omega, T_0$) could be set and fixed for the first step. After the code returns the values of the ephemeris of the EB, one could release also the astrometric parameters and run the code once again with fitting all the parameters. This strategy saves a lot of unnecessary computing time.

To conclude, using the strategy presented here and the combined method described above, one gets the satisfactory result after large number of iterations. This number and computing time is strongly dependent on the input data set (the number of observed data) and the number of parameters fitted. For the case VW Cep with the largest data set (more than 1600 data points, see below) and also with the most parameters to fit (14 in total) one reaches the solution, when the sum of squares is not changing significantly, after circa 100 000 simplex steps. This takes about one day on computer with 2 GHz processor. But it is only the illustrative example, the basic condition is the separation between the initial and the final parameters.

There were done also a few tests of the code and its ability to find the same final parameters from the different starting values of parameters. The results were satisfactory, and the code was able to find almost the same values, but it strongly depends on the separation between the initial and the final parameters. Also these tests confirmed the fact that setting the initial parameters as close to the right solution spares computing

time and also makes the final solution more reliable.

2.9 The program

The code itself was written in the Matlab language. It is available for download via the web pages <http://sirrah.troja.mff.cuni.cz/~zasche/>. The code is zipped together with the necessary routine files `*.m`, the sample input data files `*.epo` and `*.dat` and with the initial parameters file `*.in`. All of these files have to be copied into the same folder as the code itself. The sample (data files and also the code file) is for HT Vir system. The code is easily modifiable for the user. The brief manual for the users is also available.

The program was firstly designed only for the use of this thesis, so the first version was not very user-friendly, but some modifications were done for the easier use. Some comment lines were also included in the file.

In the first part of the code (lines 1 - 66) is the input. The files `*.in`, `*.dat` and `*.epo` include the input parameters, astrometry data and the minimum-time observations, respectively. The recommended data format is shown in the enclosed files. In the next lines (67 - 121) are some transformation rules and assigning the values to proper variables. Here are important the lines 74-83, where is the input of masses M_1 and M_2 and also for the parallax and its error.

In the lines 122 - 172 is the relative astrometric orbit of the binary plotted (before the computation) and saved as `HTVir-before.eps`. The next lines (173 - 299) are for the plotting of the $O - C$ diagram before the computation, saved as `HTVirOC-before.eps`.

From the line 300 to the line 681 is the body of the code, the simplex algorithm. Its initialization is in the line 327, where user could choose which mode he wants to run. There are three possibilities: 0 for only LITE, 1 for LITE together with the quadratic term (the mass transfer) and finally 2 for only the quadratic term (this possibility is not designed for the use of the combined approach). After then there are lines with the input parameters of the simplex algorithm (1.347 - 387). Here one could change the range of values of the individual parameters where the simplex works in the first run. The rest of the lines (to 1.681) is the simplex itself and has not to be modified anyway.

The lines from 683 to 1210 are for plotting the resulting $O - C$ diagrams and lines from 1211 to 1268 for plotting the resulting orbit of the binary. In the lines from 1269 to 1322 is the computation of the errors of the individual derived parameters. The lines from 1323 to 1476 are doing the output files `*.in` and `*.txt`, where the output parameters are written.

Generally, the code is ready to be used. The only modification could be the change of the names of the input and output files in the code (only find and replace the appropriate file names), and also the input masses and parallax. The main output file is `*.txt`, where are written all of the parameters with their respective errors. Also the derived quantities with the computed errors are included in the file.

The computation process of the code is the following. Run the code `*.m` in the Matlab program. After two figures ($O - C$ diagram and astrometric orbit) appear on the screen, in the main Matlab command window the program asks the user whether he wants to

compute also the quadratic term, or only the LITE. After confirmation which mode one wants to use, there appears another question about the number of iterations one wants to compute. It is up to user, but it is recommended to type only a few, because each of them could take some time. On the other hand more iterations means better precision of the result. After input these values on the screen will appear the resulting sum of square residuals of the problem, which is decreasing after each iteration. The sum of squares is divided into the two separate values, the first one from LITE and the second one from the astrometry. The most usual case is, that one of them is decreasing, while the other is increasing, but the sum of these two values has to decrease all the time. When the decreasing stops, the program terminates. A few plots will appear on the screen (these are saved as the `*.eps` files) and also the output file `*.txt` is saved to the same folder.

Chapter 3

Systems with LITE

There were a lot of studies, where many eclipsing binaries showing period variations of their period were analyzed. It was decided to present here only a few systems where LITE was not recognized till yet, or the systems where LITE was supposed to be, but another (and better) solution for the variations in the $O - C$ diagram has been found. This is not the crucial part of the thesis, and these systems are described only very briefly.

Altogether there were about 130 systems analyzed during the 3-yrs PhD study for their period changes. In about one half of these systems the LITE was proposed as a hypothetical explanation. Regrettably, the analysis was done only on the basis of their times-of-minima observations and in most of the systems any other detailed analysis was performed. This is also evident in the set of binaries presented here in this chapter.

Few of the analyzed systems were selected for the publication in various papers. Namely AD And, WY Per and V482 Per in Wolf et al. (2004), AR Aur, R CMa, FZ CMa and TX Her in Zasche (2005), OO Aql, V338 Her, T LMi, RV Lyr, TW Lac and V396 Mon in Zasche et al. (2006), EW Lyr and IV Cas in Zasche (2006) and XX Leo in Zasche & Svoboda (2006).

3.1 Individual systems under LITE analysis

All of the selected systems are Algol-type EBs, and also semidetached ones. According to the recent paper on the period changes in Algols by Hoffman et al. (2006), there could be a connection between the spectral type of the secondary component and the nature of the period changes. Systems with spectral types of secondaries earlier than F5 show $O - C$ variations, which could be caused by the magnetic activity cycles and convective envelopes. This effect was discussed by Hall (1989), Applegate (1992), Lanza et al. (1998), etc. The role of magnetic cycles on the period changes is discussed below, but due to lack of information about the systems such analysis is a difficult task. For some of the systems selected in this thesis the spectral types of secondaries are only known with a low confidence level, light-curve analysis is missing and spectroscopy has never been done.

The LITE analysis of the systems presented below in this chapter was also published in Zasche (2007).

3.2 RY Aqr

Eclipsing binary system RY Aqr (AN 125.1908, BD-11 5574, HD 203069) is an Algol-type EB. It was classified as A3 spectral type (Simbad), A8 (Popper 1989), or the most likely as a late A/early F main sequence star and an early K subgiant (Helt 1987). It is a double-lined SB (Popper 1989) with the orbital period of about 2 days. Its relative brightness is about 8.9 mag in V filter.

The variability of RY Aqr was discovered by Leavitt & Pickering (1908) and the ephemeris was firstly derived by Zinner (1913). Since then a lot of times of minima observations were obtained.

The most detailed analysis was performed by Helt 1987 on the basis of the *uvby* photoelectric photometry together with the radial velocity measurements by Popper. This analysis (using both WINK and Wilson-Devinney programs) results in a set of parameters, which reveals the nature of the system. The hotter primary has unusually low mass (about $1.27 M_{\odot}$), while the secondary K subgiant ($0.26 M_{\odot}$) has undergone a mass-loss from its initial mass of about $1.9 M_{\odot}$. There was observed also an intrinsic photometric variability which could be caused by the surface activity of the secondary (the period of this variability is close to the orbital period of the system). RY Aqr is also a member of a visual binary HU 86, but the distant body probably does not relate to the system.

There were performed a few period studies of RY Aqr (Baldwin 1974, Mallama 1980) and the most recent one by Helt (1987), who suggested that also the LITE could play a role in this system. Her supposed period of such variation (about 70 years) is different from the present one. This new analysis is based on a set of 178 times of minima published in the literature. Resultant $O - C$ diagram is plotted in Fig. 3.1 and the parameters of the third-body orbit are in Table 3.1.

The minimal mass of the third body ($M_{3,min} = M_3 \cdot \sin i_3$ if $i_3 = 90^\circ$) was calculated according to the mass function derived from the LITE hypothesis and total mass of the

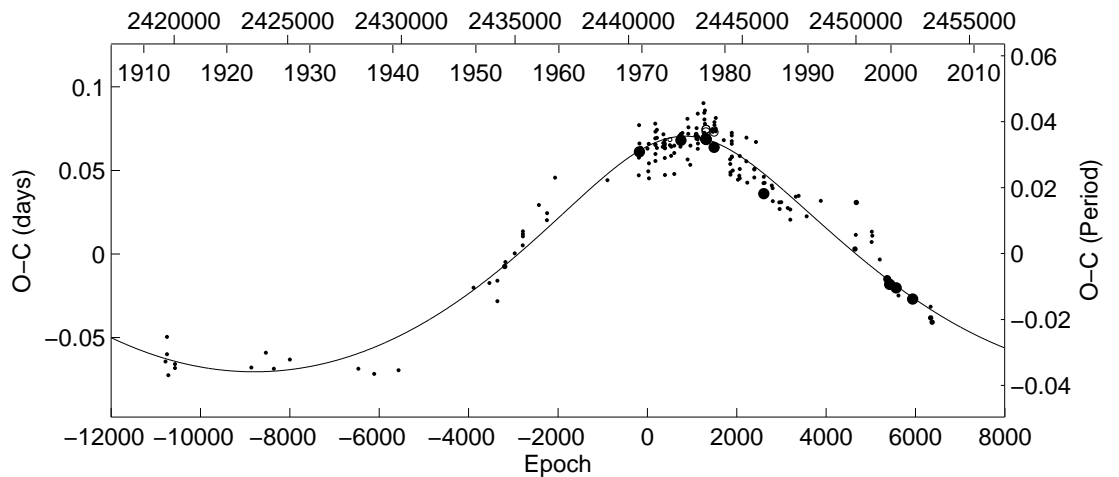


Figure 3.1: An $O - C$ diagram of RY Aqr. The individual observations are shown as dots (primary) and open circles (secondary), the small ones for visual and the large ones for CCD and photoelectric observations, bigger the symbol, bigger the weight. The curve represents the predicted LITE variation.

Table 3.1: The final results: RY Aqr.

Parameter	JD_0	P	p_3	T_0	ω	e	A	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	[M_\odot]	[M_\odot]
Value	2440824.351	1.9665990	105	2442300	88	0.35	0.070	0.165	1.02
Error	± 0.004	± 0.0000013	± 4	± 1600	± 7	± 0.05	± 0.002	± 0.006	± 0.03

eclipsing pair $M_{12} = (1.27 + 0.26) M_\odot$, see Table 3.1. Computed minimal mass about $1 M_\odot$ is rather high and such a star on the main sequence will be evident because of its luminosity. The mass $M_{3,min}$ leads to the spectral type around G3 (according to Harmanec 1988), and such a star would be more luminous than the secondary component of the EB pair. The third light in the light-curve solution by Helt (1987) was only estimated. The value about a few percent (from 2.6% in u to 5.9% in y) was adopted, but not derived. This value indicates roughly the same third mass, as was calculated from our analysis.

The radial velocities were analyzed precisely only once (Popper 1989), therefore no changes in systemic velocities are available. The systemic velocity of RY Aqr was derived to be about $-60 \text{ km} \cdot \text{s}^{-1}$, which could be caused by the motion around the common center of mass with the third component. Precise spectroscopy would probably detect the third body in the spectrum of the system.

The star has not been measured by Hipparcos, but the distance was derived from the photometry, see Helt (1987). The value $(180 \pm 10) \text{ pc}$ leads to the predicted angular separation of the third component $a = (169 \pm 10) \text{ mas}$, which was calculated using assumption $i_3 = 90^\circ$. The predicted magnitude difference is about 3 mag. The companion with such a distance and magnitude difference is detectable with the modern stellar interferometers. Only further times of minima, precise spectroscopic and photometric analysis will reveal the nature of the system.

3.3 BF CMi

BF CMi is one of the neglected eclipsing binaries, which have been observed only a few times and only very limited knowledge about it is available. Its period is about 1.18 days and the relative brightness about 10.3 mag in V filter. According to Svechnikov & Kuznetsova (1990) the star has spectral type A5+K0IV (based only on photometric indices), mass ratio 0.3, orbital inclination 79° and is a semidetached one.

Its variability was discovered by Huruhta (1979) and its designation as BF CMi was presented by Kholopov et al. (1981). But the period is still questionable. There were two unsuccessful attempts to observe the secondary minima - on the 2 March 2006 and

Table 3.2: The final results: BF CMi.

Parameter	JD_0	P	p_3	T_0	ω	e	A	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	[M_\odot]	[M_\odot]
Value	2450789.610	1.1806791	46.3	2447300	170	0.79	0.040	0.39	2.1
Error	± 0.016	± 0.0000026	± 1.2	± 200	± 21	± 0.10	± 0.018	± 0.16	± 0.9

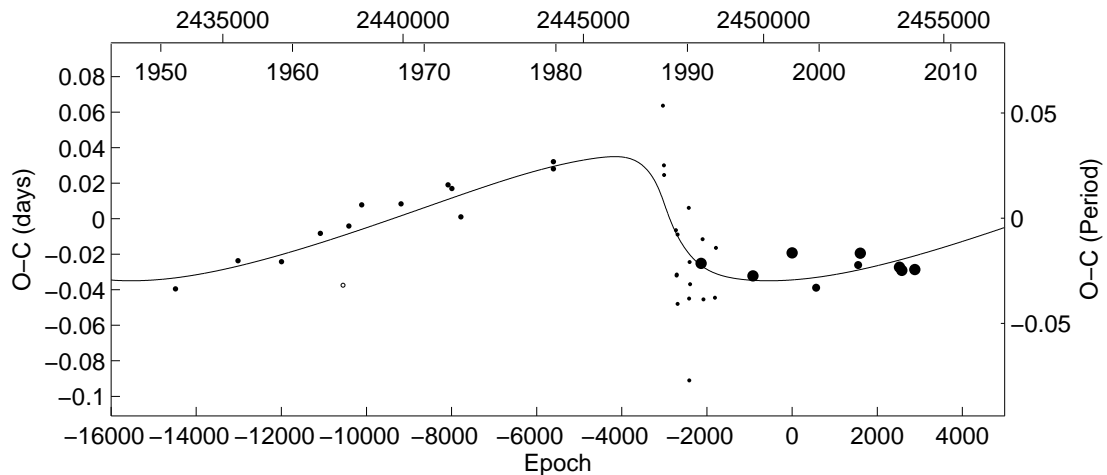


Figure 3.2: An $O - C$ diagram of BF CMi. The description is the same as in the previous figure.

4 April 2007. Also Berthold (1981) noted that no secondary minimum is observable. It means the period could be 2 times longer, about 2.36 days. Two new primary minima were observed and kindly sent by L. Šmelcer from Valašské Meziříčí observatory.

There were 39 times of minima collected from the published literature, resulting in an $O - C$ diagram shown in Fig.3.2. The parameters of LITE are in Table 3.2. As it is evident from the abrupt period jump near 1990, the eccentricity of the orbit should be rather high. Note also quite large scatter of recent photoelectric and CCD observations since 1990, which is much larger, than one could expect from these kind of measurements.

If one assume the masses of the individual components $M_1 + M_2 = (1.9 + 0.9)M_\odot$, the minimal mass of the predicted third component is about $2.1M_\odot$, which is rather high value and would dominate in the system. This hypothesis could not be proved until the detailed analysis of the system is performed. Regrettably, neither photometry nor spectroscopy was carried out. The abrupt changes in period could be also caused by the two period jumps near 1988 and 1991, produced by some mass-transfer phenomena in the system.

3.4 RW Cap

The eclipsing binary RW Cap (AN 21.1910, BD-18 5641, HD 192900) is the system with the orbital period of about 3.4 days. Its spectral type was classified as A3+A4 (according to Budding 1984) and its apparent brightness is about 10.3 mag in V filter. The depth of primary minimum is about 1.2 mag. Strömgren photometry of the system by Wolf & Kern (1983) agrees with its spectral type.

Its photometric variability was discovered by Pickering (1910) and until now there were 52 observations of times of minima obtained. Zessevich (1957) collected all available minima and proposed an abrupt period jump near 1920. After then Kreiner (1971) compiled large set of times of minima and already in this paper is evident that there could be some periodic variation in $O - C$ diagram. Regrettably during the last two decades only a few times of minima were obtained, so the LITE hypothesis is still not very conclusive.

Table 3.3: The final results: RW Cap.

Parameter	JD_0	P	p_3	T_0	ω	e	A	q	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	[10^{-10} day]	[M_\odot]	[M_\odot]
Value	2435750.857	3.3923745	80.1	2440900	0	0.23	0.130	87.4	1.91	5.9
Error	± 0.014	± 0.0000055	± 4.6	± 2800	± 42	± 0.16	± 0.011	0.2	± 0.26	± 1.0

Two of last three data points in the $O - C$ diagram are only a mean values from the automated surveys ROTSE (see Akerlof et al. 2000) and ASAS (see Pojmanski 1997). But the last one was obtained and kindly send by A.Liakos from Athens university in July 2007.

For the times of minima analysis the LITE and the quadratic term was used. This means, during the computation process, altogether 8 parameters ($JD_0, P, q, p_3, A, T_0, \omega, e$) were adjusted, resulting in a set of parameters written in Table 3.3. Because the system is semidetached, the mass-transfer hypothesis could play a role. The quadratic term coefficient $q = (87.4 \pm 0.2) \cdot 10^{-10}$ day leads to the period change about $1.88 \cdot 10^{-6}$ day/yr. From this value the conservative mass transfer rate could be derived $\dot{M} = 9.4 \cdot 10^{-8} M_\odot/\text{yr}$. Regrettably, these values cannot be proved by some other independent method.

Applying the LITE hypothesis to the system one gets the 80-years variation (see Fig.3.3), but as it is evident from the Table 3.3, the orbit is still not very well-defined and the errors of the individual parameters are large. Resulting mass function is quite high and using $M_{12} = 4.59 M_\odot$ (Brancewicz & Dworak 1980) one gets the minimal mass of the third body of about $5.9 M_\odot$. Such a body would be dominant in the light-curve solution as well as in the spectroscopic analysis. Unfortunately there were no such analysis performed. Another explanation is that the third component is also a binary.

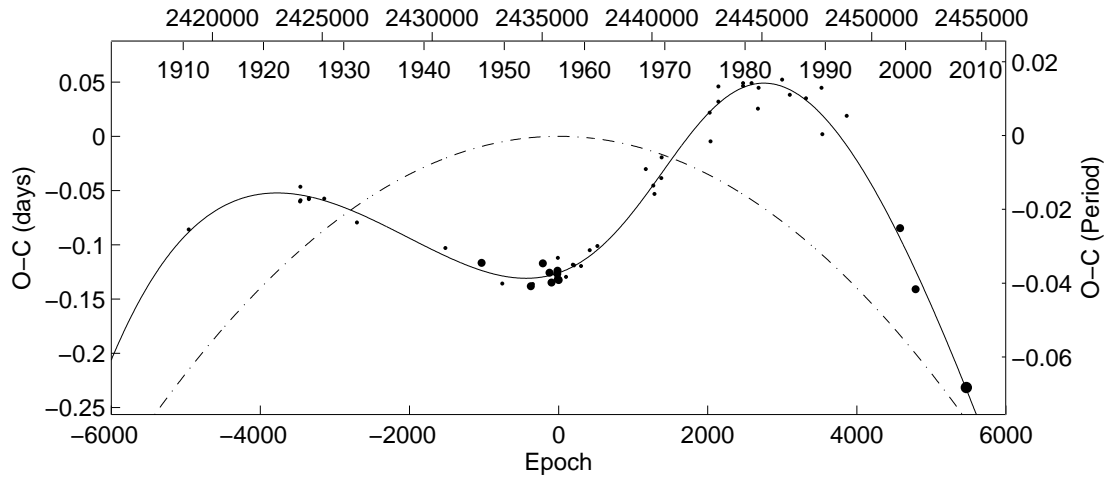
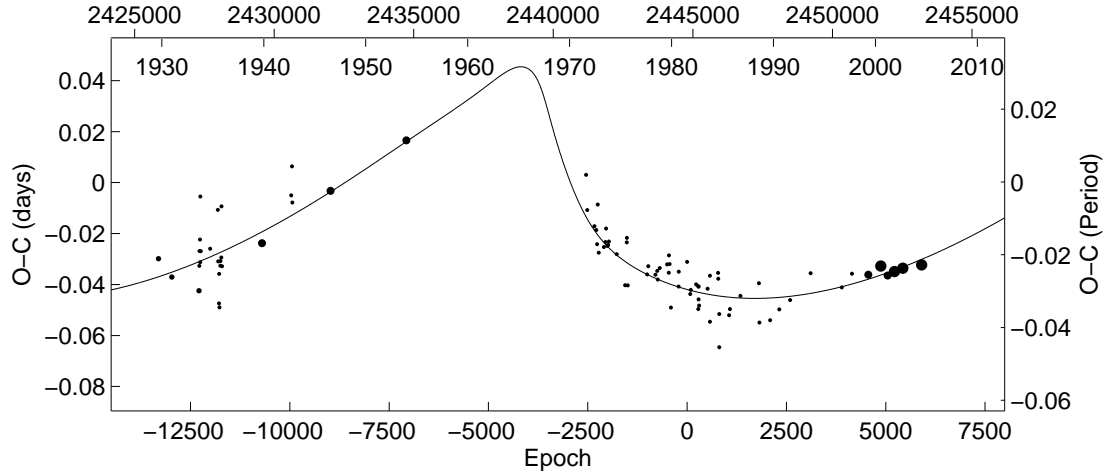


Figure 3.3: An $O - C$ diagram of RW Cap. The description is the same as in Fig.3.1, the dash-dotted line represents the quadratic term.

Figure 3.4: An $O - C$ diagram of TY Cap.

3.5 TY Cap

The next one eclipsing binary with period changes is TY Cap (AN 243.1932, BD-13 5664, HD 194168). It is an Algol-type EB with apparent brightness about 10.3 in V filter and spectral type classified as A2/3V (according to Halbedel 1984). Its orbital period is about 1.4 days.

Its photometric variability as well as its Algol-type were discovered by Hoffmeister (1933). Altogether 96 times of minimum light were carried out, only 5 data points were neglected due to their large scatter. The $O - C$ plot is in Fig.3.4, the curve represents the least-square fit with the LITE parameters written in Table 3.4.

Due to missing detailed analysis of the system, our knowledge about TY Cap is only limited. According to Brancewicz & Dworak (1980) the total mass of the EB system is $M_1 + M_2 = (2.5 + 2.06)M_\odot$. With this mass and with the parameters of the LITE from Table 3.4 one can calculate the minimal mass of the third body, which results in $2.18 M_\odot$. Unfortunately there is no spectroscopic, as well as no light-curve analysis and the star was not measured by Hipparcos, so the angular separation of the third component also cannot be derived.

The eccentricity of the LITE solution is quite high, or could be even higher, but due to lack of data points near the periastron passage this could not be proved. The next periastron passage will occur about 2035. Generally the third-body orbit is not covered sufficiently and the parameters of this predicted body have to be derived by some other independent method.

Table 3.4: The final results: TY Cap.

Parameter	JD_0	P	p_3	T_0	ω	e	A	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	$[M_\odot]$	$[M_\odot]$
Value	2444793.489	1.4234574	70.4	2439600	147	0.79	0.045	0.23	2.2
Error	± 0.006	± 0.0000009	± 8.6	± 1200	± 29	± 0.12	± 0.005	± 0.02	± 0.2

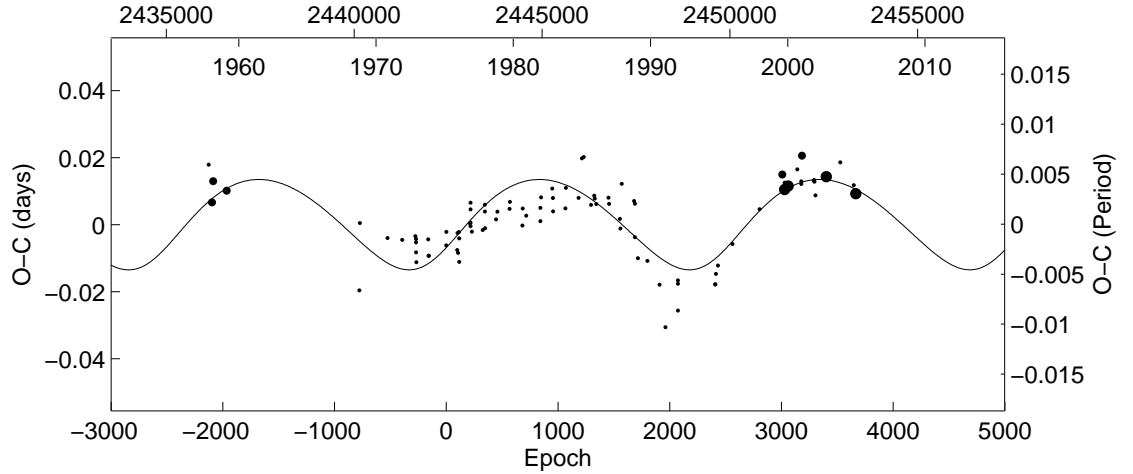


Figure 3.5: An $O - C$ diagram of SS Cet. The description is the same as in the previous figures.

3.6 SS Cet

SS Cet (BD+01 491, HD 17513) is a semidetached Algol-type EB with its apparent brightness of about 9.4 mag in V filter and spectral type classified as A0+K3III (according to Budding et al. 2004). Orbital period is about 3 days.

Its variability was discovered by Hoffmeister (1934a). The most detailed analysis was performed by Narasaki & Etzel (1994) on the basis of their *BVRI* photoelectric photometry and spectroscopy. This study results in a semidetached system, with no peculiarities in the light curve, $M_1 = 2.15M_\odot$, $M_2 = 0.6M_\odot$, and the spectroscopic observations indicate that no circumstellar matter is presented in the system. On the other hand the spectroscopic analysis by Vesper et al. (2001) presents an evidence for the mass transfer in the binary because of the behavior of the $H\alpha$ emission. The mass transfer was not recognized in our current analysis of the times of minima.

Since its discovery there were 111 minima observations obtained, but only 95 were used for the period analysis, because of the large scatter of the first ones from 1930's. There could be a period jump near 1950, this is the reason why only the recent data were analyzed. The period changes were firstly noted by Kreiner (1971), but only with a small set of times of minima, displaying the steady increase. The final LITE curve is in Fig. 3.5 and the resultant parameters in Table 3.5.

Using the mass of SS Cet derived by Narasaki & Etzel (1994) $M_{12} = 2.75M_\odot$, one gets the minimal mass of the third component about $0.72M_\odot$. The predicted value of the third light is only about 1 % and the third component would be also similar to the secondary

Table 3.5: The final results: SS Cet.

Parameter	JD_0	P	p_3	T_0	ω	e	A	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	[M_\odot]	[M_\odot]
Value	2442451.330	2.9739737	20.4	2449500	304	0.21	0.014	0.031	0.75
Error	± 0.002	± 0.0000007	± 0.5	± 900	± 73	± 0.09	± 0.002	± 0.002	± 0.13

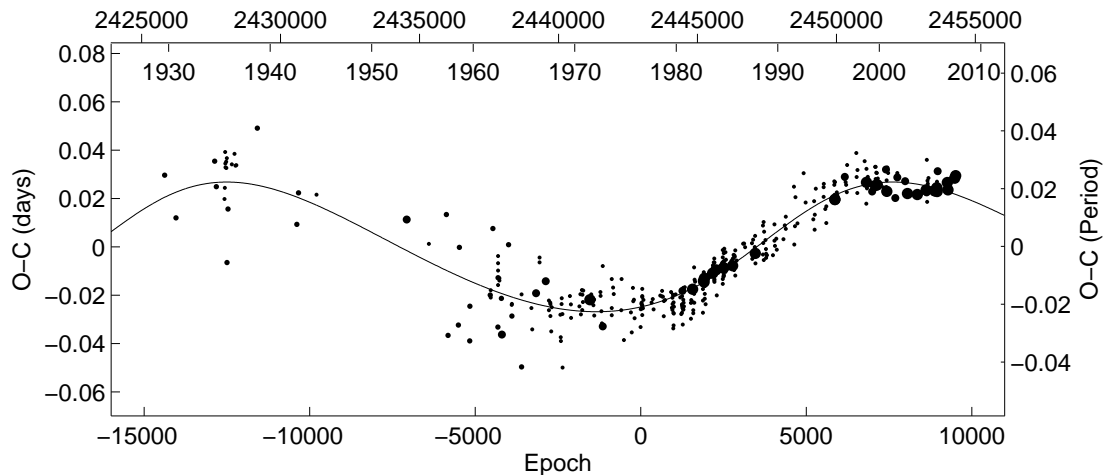


Figure 3.6: An $O - C$ diagram of TY Del. The description is the same as in the previous figures.

component in the spectra. Detect the third body in the light curve as well as in the spectra is a difficult task. The distance to the system was derived by Narasaki & Etzel (1994), resulting in $d = 486$ pc. According to this value and derived parameters of the third body, one could calculate the predicted angular separation of the third component about only 23 mas and magnitude difference about 5 mag. Such a large magnitude difference and low separation is hardly detectable, and the third body remains undetectable also by interferometry.

3.7 TY Del

Another EB with apparently variable period is TY Del (AN 141.1935, BD+12 4539), spectrum classified as B9+G0IV (Hoffman et al. 2006) and relative brightness about 10.1 mag in V filter. There is a consensus about the spectral types of the components of TY Del, but there is a difference between the masses. Brancewicz & Dworak (1980), and after then also Budding (1984) and Budding et al. (2004) have presented the masses $M_1 = 5M_\odot$, $M_2 = 2M_\odot$, while Svechnikov & Kuznetsova (1990) presented $M_1 = 2.8M_\odot$, $M_2 = 0.84M_\odot$, what is more likely to the proposed spectral types.

The star was discovered to be a variable by Hoffmeister (1935). There were only one attempt to observe the whole light curve of TY Del photoelectrically by Faulkner (1983), unfortunately only about a half of the curve was observed. No analysis of these data was carried out. The star was also studied by Cook (1993) on the basis of his

Table 3.6: The final results: TY Del.

Parameter	JD_0	P	p_3	T_0	ω	e	A	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	$[M_\odot]$	$[M_\odot]$
Value	2442959.471	1.1911264	64.9	2449200	38	0.22	0.027	0.025	0.79
Error	± 0.001	± 0.0000002	± 2.3	± 1000	± 18	± 0.06	± 0.002	± 0.001	± 0.07

visual observations for the long-time scale intrinsic variations, but the results are not very conclusive.

The spectroscopic observations in $H\alpha$ were done by Vesper et al. (2001). They conclude that there is no activity in $H\alpha$ and no evidence for the mass transfer structures was found in this system.

Altogether 370 times of minima were collected, from which only 5 were omitted due to their large scatter. One period of the third body is already sufficiently covered by data points, see Fig. 3.6, but further observations are still needed. The last one data point was observed at Ondřejov observatory. From the LITE parameters (see Table 3.6) and with the approximate masses of the individual components of the eclipsing binary $M_1 = 2.8 M_\odot$, $M_2 = 0.84 M_\odot$ (Svechnikov & Kuznetsova 1990) one is able to derive the minimal mass of the third component $M_{3,min} = 0.67 M_\odot$. Due to lack of any other observations also this hypothesis cannot be proved. The spectral types and masses were derived only on the basis of the photometric indices and are not very conclusive. The spectroscopic analysis, as well as the analysis of the light curve of the system is needed, but the third light is undetectable in the light-curve solution. Regrettably the star was not measured by Hipparcos, so the distance is not known and one cannot derive the predicted angular separation of the third component. As one can see, there is some additional variation besides LITE, which is not strictly periodic, see Section 3.14 for details.

3.8 RR Dra

Another eclipsing binary which exhibits apparent period changes is RR Dra (AN 188.1904). It is an Algol-type semidetached binary, relative brightness about 9.8 mag in V, spectrum classified as A2+G8IV (Svechnikov & Kuznetsova 1990), while Yoon et al. (1994) classified a little bit later spectral type of secondary K0. Svechnikov & Kuznetsova (1990) presented the masses $M_1 = 2.15M_\odot$ and $M_2 = 0.6M_\odot$. The primary minimum is very deep, about 3.5 mag and the orbital period is about 2.8 days.

The star was discovered to be a variable by Miss Ceraski (1905). The minimum is so deep that also visual observers could provide reliable observations. That is the reason why most of the collected times of minima are the visual ones (193 out of 219). Kreiner (1971) collected all available minima for the period analysis. The long-time increase of the period is evident from his $O - C$ diagram (due to the mass transfer between the components?). The most recent period study of this system was performed by Qian et al. (2002), who considered (besides the mass transfer) the abrupt period jumps - altogether 8 jumps were introduced to describe the $O - C$ diagram in detail. Almost the same goodness of fit could be reached by applying the LITE hypothesis besides the mass transfer.

Table 3.7: The final results: RR Dra.

Parameter	JD_0	P	p_3	T_0	ω	e	A	q	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	[10^{-10} day]	[M_\odot]	[M_\odot]
Value	2434913.728	2.8312140	84.3	2450100	110	0.50	0.073	-126.2	0.300	1.85
Error	± 0.022	± 0.0000053	± 0.6	± 400	± 4	± 0.03	± 0.002	0.2	± 0.002	± 0.09

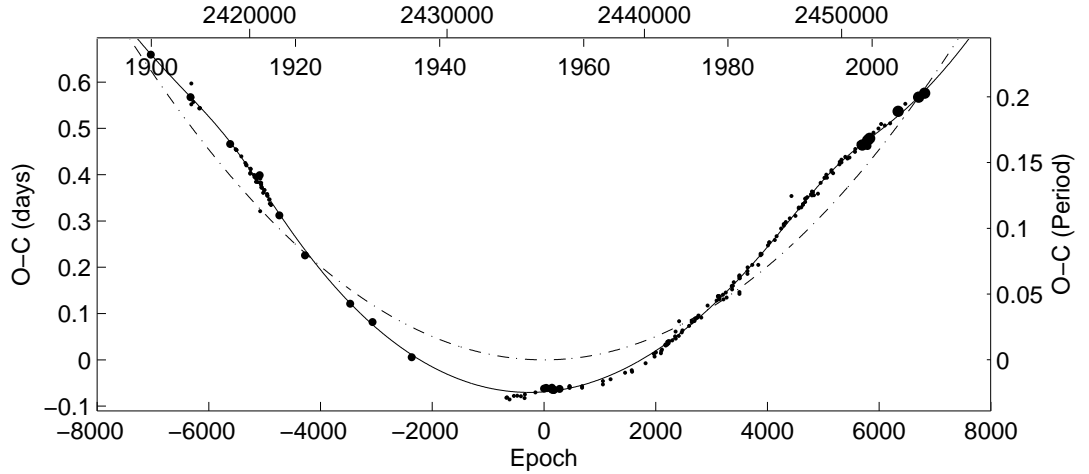


Figure 3.7: An $O - C$ diagram of RR Dra. For the plot where the quadratic term was subtracted see Fig.3.8.

Altogether 219 times of minima were used for the analysis. One new observation of minimum was obtained by M.Wolf at Ondřejov observatory. The $O - C$ plot is in Fig.3.7, where LITE and the quadratic term were plotted together. In the next figure only LITE is shown, see Fig.3.8. As one can see, the period increase is very rapid, and the amplitude of LITE is still quite high. This leads to the relatively high mass function, which results in high predicted minimal mass $M_{3,min} = 1.85M_{\odot}$. This is larger than the secondary and in the light-curve solution as well as in the spectrum will be probably observable. Regrettably no such analysis was performed.

The quadratic term coefficient $q = (126.2 \pm 0.1) \cdot 10^{-10}$ day leads to the period change about $3.26 \cdot 10^{-6}$ day/yr. From this value the conservative mass transfer rate could be derived $\dot{M} = 3.5 \cdot 10^{-7} M_{\odot}/\text{yr}$. This relatively high value of mass transfer rate arises from the very rapid period change, which was attributed to the quadratic ephemeris. The

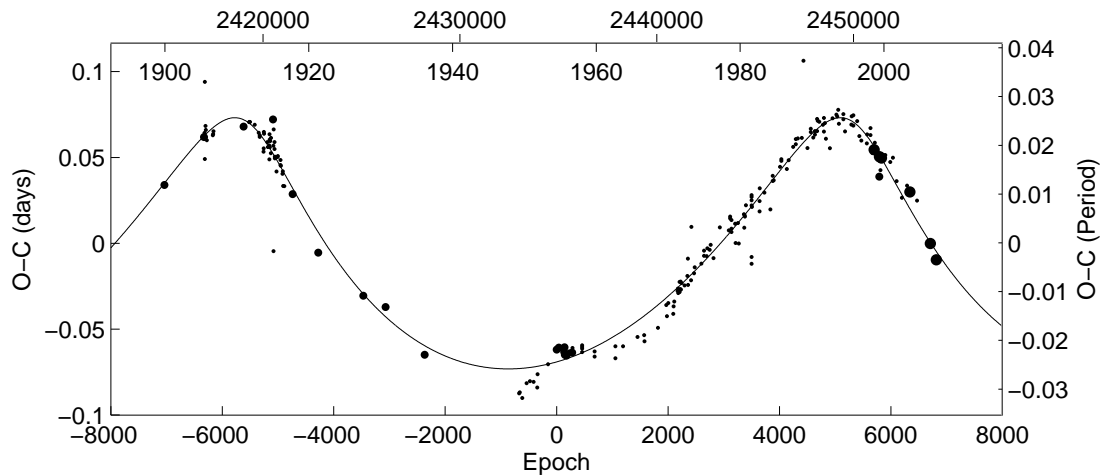


Figure 3.8: An $O - C$ diagram of RR Dra after subtraction of the quadratic term.

spectroscopic observations during the primary eclipse made by Kaitchuck et al. (1985) indicate a possible presence of a transient accretion disc in the system. The presence of such disc also supports the hypothesis of mass transfer in the system.

For the estimation of the angular separation of the third body and the astrometric confirmation of the LITE hypothesis the distance to the system has to be known. The star was not measured by Hipparcos and the distance is not known precisely. The only information about the distance is from Kharchenko (2001), where is introduced surprisingly inaccurate value of the parallax $\pi = (0.40 \pm 11.50)$ mas. Distance with such a large error is useless for the estimation of the angular separation of the predicted component.

3.9 TZ Eri

The system TZ Eri (AN 40.1929, BD-06 880) is the EB with the orbital period of about 2.6 days, apparent brightness of about 9.7 mag in V filter. It has a deep primary minimum (about 2.8 mag), so the visual observations could be also reliable.

Its variability was discovered by Hoffmeister (1929), who also recognized the system to be an Algol-type. The spectral type was firstly classified by Miss Cannon (1934) as F. Later the spectrum was re-classified as A5/6 V (primary) and K0/1 III (secondary) by Barblan et al. (1998). In this later paper the light-curve observations in the Geneva 7-colour photometric system were analyzed together with the radial-velocity curves of both components. Wilson-Devinney code was used, resulting in a set of parameters describing both components. For our analysis are the most important the masses, $M_1 = 1.97 M_\odot$ and $M_2 = 0.37 M_\odot$.

There were also several studies about the presence of the accretion disc in the system (e.g. Kaitchuck & Honeycutt 1982, Kaitchuk & Park 1988, Vesper et al. 2001). This disc as well as mass transfer from the secondary to the primary is in agreement with our result about the increasing orbital period (see below). The system was also included in the sample of Algol-type binaries with the radio emission (Umana et al. 1998). The star was

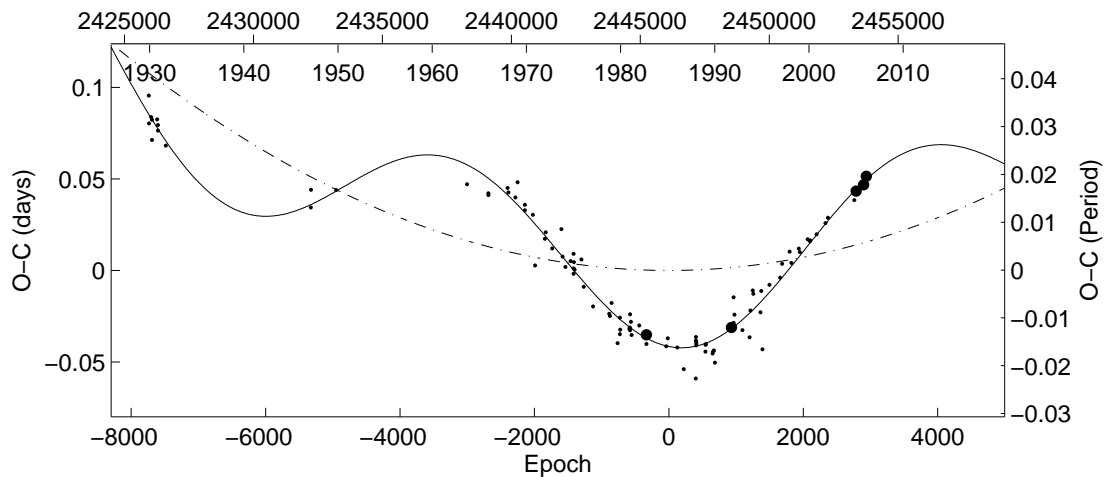


Figure 3.9: An $O - C$ diagram of TZ Eri.

Table 3.8: The final results: TZ Eri.

Parameter	JD_0	P	p_3	T_0	ω	e	A	q	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	[10^{-10} day]	[M_\odot]	[M_\odot]
Value	2446109.730	2.6061129	48.8	2451100	0	0.01	0.042	-18.0	0.165	1.3
Error	± 0.009	± 0.0000034	± 6.8	± 2300	± 40	± 0.10	± 0.014	0.2	± 0.013	± 0.1

also investigated according to the possible connection between the orbital and pulsational periods, see Soydugan et al. (2006).

The analysis of the long-term period changes was done with a set of 108 observations (mostly the visual ones). The resultant $O - C$ diagram is in Fig.3.9 and the parameters of the predicted LITE are in Table 3.8. The minimal mass of the third component results in $M_{3,min} = 1.3 M_\odot$, or the spectral type F6 (according to Harmanec 1988). Such a body could be evident in the light-curve solution as well as in the spectra of TZ Eri. Regrettably there were no attempt to detect such a body during the detailed analysis by Barblan et al. (1998). The long-term period increase is due to the mass transfer from the secondary, with the conservative mass-transfer rate $\dot{M} = 6.2 \cdot 10^{-8} M_\odot/\text{yr}$.

Despite the fact the star was not observed by Hipparcos, Barblan et al. (1998) estimated the photometric distance to $d = (270 \pm 12)$ pc. Assuming the coplanar orbit of the third component, then $M_3 = M_{3,min}$ and one could calculate the predicted angular separation of the third body to $a = 77$ mas and magnitude difference about 1.7 mag. Such a component is detectable with the modern stellar interferometers.

3.10 RV Per

The system RV Per (AN 61.1905, BD+33 805, HD 279552) is the EB with the orbital period of about 2 days, spectral type classified as A2+G7IV, according to Svechnikov & Kuznetsova (1990). Also this star shows deep primary minimum, about 2.4 mag.

The star was discovered to be a variable by Blažko (1907). Since then only a few papers on this star were published, so our knowledge about the system is very limited. According to Brancewicz & Dworak (1980) the masses of the individual components are $M_1 = 3.04 M_\odot$ and $M_2 = 0.46 M_\odot$. The attempts to prove the existence of the accretion disc in the system (Kaitchuck et al. 1985) were not successful, as well as the presence of the pulsating component in the system was not confirmed (Kim et al. 2003).

The period changes were firstly studied by Wood (1950), but the data set was not sufficient to do any satisfactory conclusions. In the present thesis the data set consists of 146 times-of-minima observations (see Fig.3.10). One new minimum was observed by M.Zejda. The parameters of LITE are in Table 3.9. As one can see, the period of the third

Table 3.9: The final results: RV Per.

Parameter	JD_0	P	p_3	T_0	ω	e	A	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	[M_\odot]	[M_\odot]
Value	2442046.920	1.9734888	99.8	2431600	210	0.79	0.017	0.007	0.47
Error	± 0.002	± 0.0000003	± 18.8	± 7200	± 44	± 0.46	± 0.003	± 0.002	± 0.07

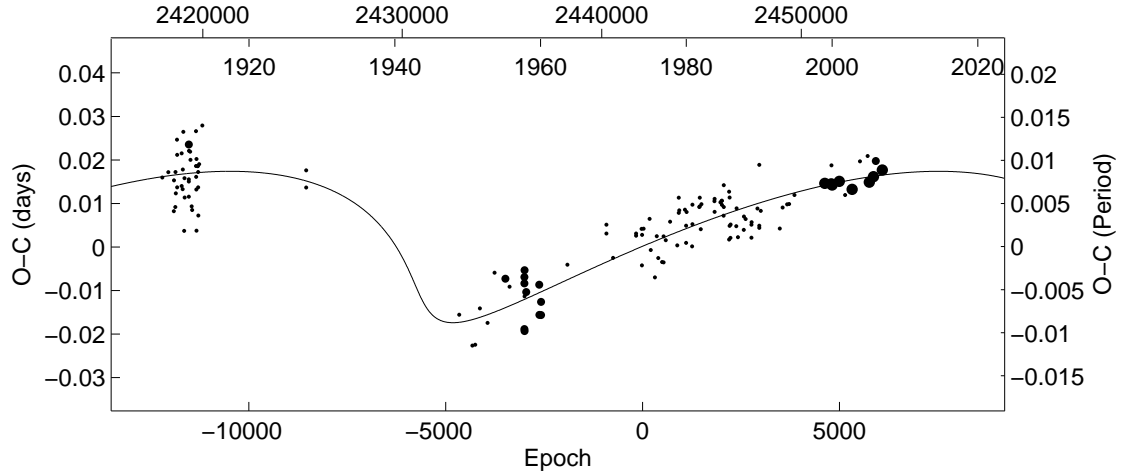


Figure 3.10: An $O - C$ diagram of RV Per.

body is not covered by observations yet and the errors of the individual parameters are high. The value of eccentricity could be even higher, but there are no observations near the periastron passage, and the next one is predicted to occur near 2040. Only further observations would confirm or reject the third-body hypothesis.

3.11 UZ Sge

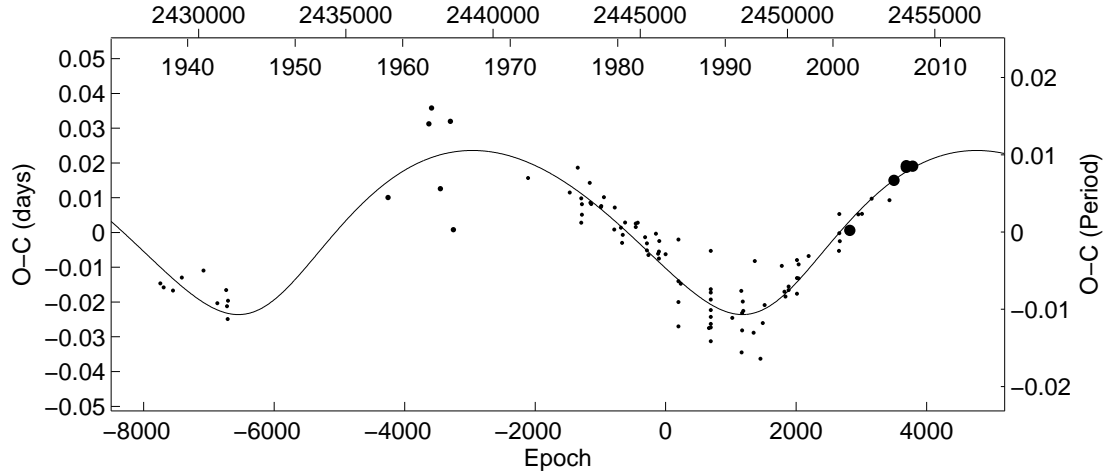
The Algol-type EB system UZ Sge (AN 435.1936) has the orbital period of about 2.2 days and spectral type classified as A3V+G0IV (Svechnikov & Kuznetsova 1990).

Its photometric variability was discovered by Guthnick & Schneller (1939). Since then there was no attempt to do any detailed analysis, neither the photometric nor the spectroscopic one. The only spectroscopic observation was done by Halbedel (1984) for derivation of the spectral type of primary component.

Altogether 122 measurements of times of minima were found in literature, but 14 observations were neglected. Four new observations of minima were obtained (two of them by L. Šmelcer, one by M. Wolf and one by author). If the masses of the individual components were taken from Svechnikov & Kuznetsova (1990), $M_1 = 2.05 M_\odot$ and $M_2 = 0.29 M_\odot$, then the minimal mass of the third component results in $M_{3,min} = 0.65 M_\odot$. But due to absence of any detailed analysis of this system, this value cannot be proved.

Table 3.10: The final results: UZ Sge.

Parameter	JD_0	P	p_3	T_0	ω	e	A	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	$[M_\odot]$	$[M_\odot]$
Value	2445861.420	2.2157425	47.0	2449200	294	0.28	0.023	0.031	0.65
Error	± 0.002	± 0.0000007	± 2.4	± 2000	± 47	± 0.13	± 0.003	± 0.002	± 0.05

Figure 3.11: An $O - C$ diagram of UZ Sge.

3.12 BO Vul

The last EB system in this thesis which shows long-term period changes is BO Vul (AN 125.1935, HD 345287). It is the eclipsing binary with the orbital period of about 1.9 days, apparent brightness 10 mag, depth of primary minimum about 1.6 mag and spectral type F0+G0IV (Svechnikov & Kuznetsova 1990).

The star was observed to be a variable by Hoffmeister (1935) and the first ephemeris were presented by Guthnick & Prager (1936). The first observation of the whole light curve was carried out by Nassau (1939), where also a brief analysis of the system was presented. Since then there was no detailed analysis of the system performed.

The changes of its period were firstly mentioned by Ahnert (1973). After then also Baldwin (1996) published new revised elements for BO Vul, but without any interpretation of the period changes.

Altogether 390 times of minima were collected, but only 360 were used for this analysis. One new minimum was observed by M.Wolf. For the final results see Fig.3.12 and Table 3.11. If the masses from Svechnikov & Kuznetsova (1990) were taken, $M_1 = 1.45 M_\odot$ and $M_2 = 0.64 M_\odot$, then the minimal mass of the third component results in $0.73 M_\odot$. But as in the previous cases, there is no detailed analysis, which could prove this result. The quadratic term leads to the conservative mass-transfer rate of about $\dot{M} = 1.3 \cdot 10^{-7} M_\odot/\text{yr}$.

One can also see some additional non-periodic changes, which are evident since 1970's and which could not be described by applying only LITE and the quadratic term. The amplitude of these variations is smaller than the amplitude of LITE, but one cannot doubt

Table 3.11: The final results: BO Vul.

Parameter	JD_0	P	p_3	T_0	ω	e	A	q	$f(M_3)$	$M_{3,min}$
Unit	[HJD]	[day]	[yr]	[HJD]	[deg]		[day]	[10^{-10} day]	[M_\odot]	[M_\odot]
Value	2441163.509	1.9458790	42.2	2446900	0	0.33	0.024	25.5	0.049	0.73
Error	± 0.002	± 0.0000006	± 1.3	± 1800	± 18	± 0.10	± 0.002	0.1	± 0.003	± 0.04

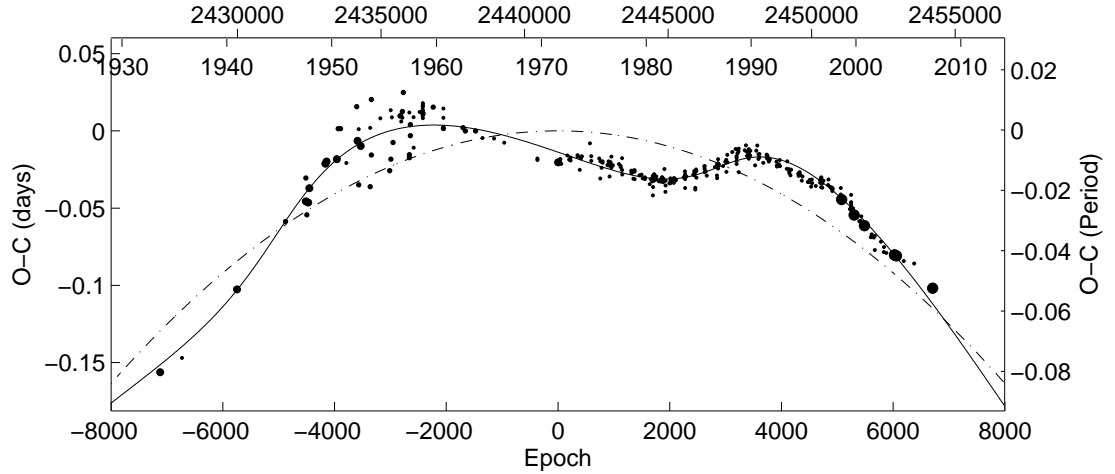


Figure 3.12: An $O - C$ diagram of BO Vul.

about their presence nowadays. These could be caused by the abrupt period changes, or due to magnetic activity cycles presented in the system. See the next section for the brief analysis.

According to Brancewicz & Dworak (1980) the distance to the system is about 233 pc, but the star was not measured by Hipparcos, so this value is not very reliable (there is no information about the error of this value). From the distance one could estimate the predicted angular separation of the third component, resulting in 63 mas, and magnitude difference about 3.4 mag. Such body is hardly detectable within the modern stellar interferometers.

3.13 Alternative explanation

One can also see some additional non-periodic variations in some of the $O - C$ diagrams, which could not be described by applying only the LITE hypothesis. In Figs. 3.13 there are shown four cases with the most evident variations. The amplitudes of these variations are usually about 10 minutes in the $O - C$ diagram and are not strictly periodic (the "periods" are from 5 to 20 years). This could be caused by the presence of the stellar convection zones and magnetic activity cycles in an agreement with so-called Applegate's mechanism, see e.g. Applegate (1992), Lanza et al. (1998), or Hoffman et al. (2006). The effect could play a role, because the spectral types of the secondary components in most of the systems are later than F5 (see Zavala et al. (2002) for a detailed analysis). This explanation would clarify the non-periodicity and the changes in amplitude of such variation, as well as why in some binaries this phenomena is presented, while in some others not.

Due to missing information about the properties of the eclipsing components in most of the systems one also cannot estimate the predicted variation of the quadruple moment

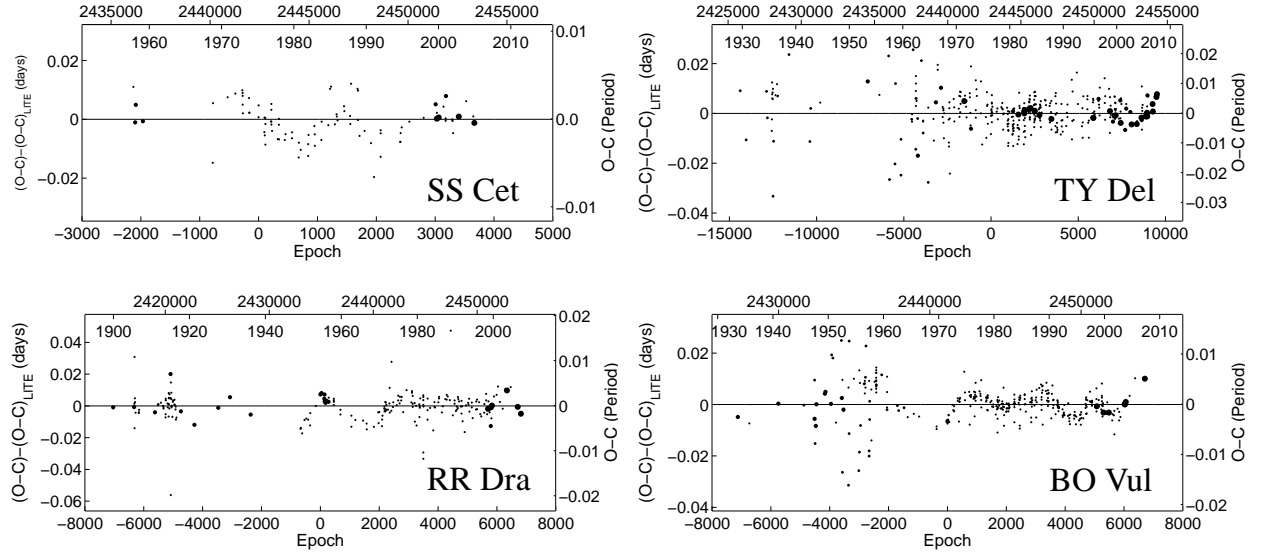


Figure 3.13: The $O - C$ diagrams of four systems after subtraction of the LITE and the quadratic term (cases RR Dra and BO Vul). The additional variation is clearly visible.

ΔQ , which causes the period variations. This value could be computed from the equation

$$\frac{\Delta P}{P} = -9 \frac{\Delta Q}{M a^2},$$

where P is the orbital period of the system, M is a mass of the star and a is the separation between the components, see Lanza & Rodonò (2002). ΔP is the amplitude of the period oscillation and could be computed from the LITE parameters from equation

$$\Delta P = A \cdot \sqrt{2[1 - \cos(\frac{2\pi P}{p_3})]},$$

see Rovithis-Livaniou et al. (2000). The typical values of ΔQ are of the order of $10^{51} - 10^{52} \text{g} \cdot \text{cm}^2$ (Lanza & Rodonò 1999).

Due to missing light-curve and radial-velocity curve analysis, the value a is missing. From a few cases where this value is known (RY Aqr, SS Cet and TZ Eri) only RY Aqr does not satisfy the condition about the limits for ΔQ (being about 10times lower). This result does not indicate that the magnetic activity cycles are not presented in this system, but only the fact that this effect cannot be used as an alternative explanation of the period changes. The effect could be presented as an additional to the light-time effect and describe the non-periodic variations (shown in Figs. 3.13 after subtraction of LITE).

To conclude, for the better description of the observed period variations of these systems, the magnetic activity cycles could be presented together with the LITE. On the other hand one has to take into consideration that the spectral types of most of these binaries were derived only on the basis of their photometric indices (Svechnikov & Kuznetsova 1990) and are not very reliable.

3.14 Brief summary

Eleven Algol-type semidetached eclipsing binaries were analyzed for the presence of LITE on the basis of their $O - C$ diagram analysis and the times-of-minima variations. A few new observations of these systems were obtained. All of the systems above show apparent changes of their orbital periods, which could be explained as a result of orbiting the EB around the common center of mass with the third component.

Such a variation has usually the period in order of decades. The light-time effect was applied as a main cause of these changes (as one can see from Figs.3.1–3.12). In four cases (RW Cap, RR Dra, TZ Eri and BO Vul) also the quadratic term in the light elements was used. This could be described as a mass transfer between the components, which could play a role, because all of the systems are semidetached ones. In some cases also the proof of presence of mass-transfer structures or accretion discs were revealed by spectroscopy.

Regrettably in most of the systems any detailed analysis (neither the photometric nor the spectroscopic) was carried out. The spectral types and the masses of the individual components in most of the systems are only approximate, so the parameters of the predicted third bodies are also affected by relatively large errors. Due to missing information about the distances to most of these binaries also the prediction about the angular separation could not be done. As one can see, only further detailed photometric, as well as spectroscopic and interferometric analysis would reveal the nature of the system and confirm or refuse the third-body hypothesis.

Chapter 4

Systems with combined LITE and astrometry

The crucial part of this thesis was the analysis of the systems, where the EB pair is a component of spatially resolved binary. Despite increasing number of the visual as well as eclipsing binaries, the intersection of these two sets is still only very limited.

Find an appropriate candidate for this analysis turned out to be quite difficult. One of the problems was the data set and its quality. Such systems have to satisfy the following adopted conditions: 1. More than 10 times of minima and more than 10 astrometric observations are available. 2. The observed range of the position angle in the astrometric measurements is larger than 10° . The limit for the number of data points was accepted because of the number of parameters, which have to be found. There were 5 parameters for LITE and 2 ephemeris, or 7 for astrometry, it means at least 7 data points (in both methods) is the absolute minimum for the analysis. And the limit for the range of data points was the accuracy of the fit. This means fitting the linear part of the astrometric orbit (or the $O - C$ diagram) is useless for the parameter determination.

The astrometric measurements were adopted from "The Washington Double Star Catalog" (hereafter WDS ¹), which incorporates a huge database of astrometric observations, but only a small one about the properties of the individual components in these systems. There was a problem with identifying the eclipsing binaries in them. Altogether more than 13800 systems in the northern and southern sky have been inspected. This was the number of systems in WDS with 10 or more astrometric observations. From this large number of stars only 31 were eclipsing binaries (according to Simbad database). And from this 31 stars in most cases there were no or only a little data set in times of minima. Systems with larger times-of-minima data sets which have been analyzed are presented below in this chapter. In the next section is the brief survey of another systems.

In a few cases the astrometry and the behaviour of times of minimum light were studied, but these two approaches were usually analyzed separately. Such systems are for example 44 Boo, QZ Car, SZ Cam, or GT Mus (besides the systems mentioned and analyzed below). The coverage of the astrometric orbit is very poor for some of them. For SZ Cam only a few usable astrometric observations were obtained, but the LITE

¹[http : //ad.usno.navy.mil/wds/](http://ad.usno.navy.mil/wds/)

is well-defined and also the third light in the light-curve solution was detected (Lorenz et al. 1998). QZ Car is a more complicated, probably quintuple system - the bright component of the visual binary consists of two eclipsing pairs ($P = 20.7$ d and 6.0 d). There are also only a few usable astrometric measurements. Also GT Mus is a quadruple system, consisting of an eclipsing and RS CVn component. In many other cases, only measurements of the times of minima are available, without astrometry. For some others, astrometry without photometry, is only available. Another systems where the astrometric observations were obtained and the LITE is observable or expected are listed in Mayer (2004).

The only paper on combining the two different approaches (LITE and astrometry) into one joint solution is that by Ribas et al. (2002), where the similar method (but not the same) as described in this thesis was applied to the system R CMa, but where only a small arc of the astrometric orbit was available. Besides the astrometry and LITE also the proper motion on the long orbit was analyzed. On the other hand one has to note, that in Ribas et al. (2002) the complete astrometric parameters (with proper motions, parallax, etc.) were used, while in this thesis only a relative astrometry of the distant body relative to the eclipsing pair was analyzed. From this point of view such an approach to the combination of LITE and astrometry is unique and has never been published before. Generally, such a combined approach is potentially very powerful, especially in upcoming astrometric and photometric space missions.

Three of the systems presented below in this chapter (namely VW Cep, ζ Phe and HT Vir) were selected for the publication, see Zasche & Wolf (2007) (in print)

4.1 QS Aql

The first investigated system is QS Aql (KUI 93, HD 185936, HR 7486, HIP 96840). It is the Algol-type eclipsing, and also spectroscopic, binary with a period of about 2.5 days. Its apparent brightness is about 6.0 mag and the spectral type was classified as B5V (according to Holmgren 1987).

The star was recognized to be a variable by Dr. Plaskett from The Dominion Astrophysical Observatory from the photographic plates taken in 1924 and 1925 (Millman 1928). The first photometric observations were obtained by Guthnick (1931). Surprisingly, only 17 times of minima were recorded since then. This is probably due to the relatively high brightness of the object, which would saturate most of the telescopes with the CCD detectors (the last one is from Hipparcos).

Guthnick (1931) recognized the eclipsing nature of the star. Some 40 years later, Knipe (1971) discovered a rapid period change, which occurred at about 1964 (his suggestion) and was caused by the periastron passage in the wide orbit around the barycenter. The period change was so rapid that the eccentricity of the wide orbit must be very high. Unfortunately, during the last decade no minimum time was obtained, the last one is more than 15 years old.

The first astrometric observations were secured more than 50 years ago, but their accuracy is questionable. Because both visual components are similarly bright, there could

Table 4.1: The parameters of QS Aql.

Parameter	JD_0	P	p_3	A	T_0	ω	e	i	Ω	$f(M_3)$	M_3
Unit	[day]	[day]	[yr]	[day]	[day]	[deg]	-	[deg]	[deg]	[M_\odot]	[M_\odot]
Value	2440443.4680	2.51330731	82.0	0.0516	2437313	329.8	0.940	21.0	163.7	0.535	16.5
Error	0.0003	0.00000098	1.9	0.0039	24	9.6	0.008	9.9	1.5	0.425	$^{+77.0}_{-12.8}$

be a confusion in the identification of the primary and the secondary, and some measurements may be shifted for about 180° in the position angle. Especially due to this reason, we have neglected all measurements obtained before 1975. More recent data are more reliable and more precise (since 1976 the observations are mostly speckle interferometric).

The only paper considering the astrometry together with the LITE was published by Mayer (2004). The system QS Aql is presented there as one of the systems where LITE could be observed together with the astrometric orbit. Also the warning regarding the quality of the old data and their 180° ambiguity is given there.

In Table 4.2, the observed times of minima of QS Aql and the corresponding epochs relative to the ephemeris given in Table 4.1 can be found. The algorithm presented in the introduction was used to analyze this system combining the astrometry and the times-of-minima analysis. The resultant parameters of the distant-body orbit are presented in Table 4.1. In Figs. 4.1 and 4.2, the $O-C$ diagram of the times of minima, and the astrometric orbit are shown, respectively. The curves in both figures show the model fit corresponding to the resultant parameters given in Table 4.1.

The spectrum of QS Aql was firstly classified as B3 (Millman 1928), but already the next spectroscopic analysis by (Hill 1931) indicates a later spectral type B5. In this latter paper also the first spectroscopic orbit was calculated. The analysis of this SB1-type binary results in $e = 0.056 \pm 0.027$, $K = (47.31 \pm 1.31) \text{ km} \cdot \text{s}^{-1}$, $v_\gamma = (-14.21 \pm 0.98) \text{ km} \cdot \text{s}^{-1}$. After then a few spectroscopic investigations of this binary

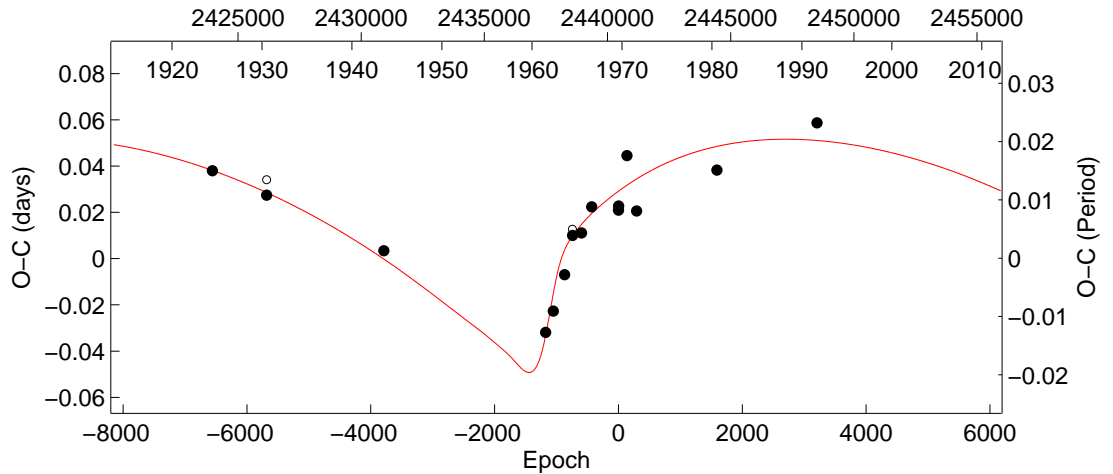


Figure 4.1: An $O-C$ diagram of QS Aql. The individual observations are shown as dots (primary) and open circles (secondary) and the curve represents the predicted LITE. All of the measurements are photoelectric or CCD ones.

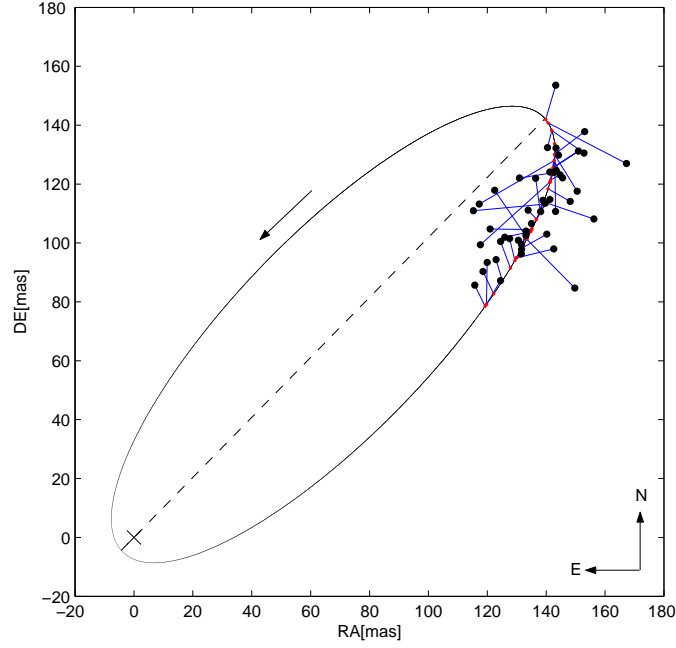


Figure 4.2: A relative astrometric orbit of a system QS Aql. Older measurements were neglected. The points represent individual observations (black dots), while the solid curve corresponds to the solution described in the text and parameters in Table 4.1. The straight lines connect individual observations with their expected positions on the fitted orbit (red dots). The cross indicates the position of the eclipsing binary on the sky, the arrow indicates the direction of the movement on the third-body orbit, and the dashed line represents the line of apsides.

were carried out. The systemic velocities were derived: $-4.93 \text{ km} \cdot \text{s}^{-1}$ (low confidence level, no spectroscopic solution, just an estimated mean velocity of the system, Millman 1928), $-13 \text{ km} \cdot \text{s}^{-1}$ (Lucy & Sweeney 1971), $-15.9 \text{ km} \cdot \text{s}^{-1}$ (Batten et al. 1978), and the most recent one by Holmgren (1987) results in $(-14.8 \pm 0.2) \text{ km} \cdot \text{s}^{-1}$. As one can see from Fig.4.3, the observed systemic velocities are almost constant over the period of the third body, which seems unlikely. But it is necessary to take into consideration the error bars (which are not known for some of these points) and also the very rapid change in v_γ near the periastron and almost constant velocity for a decades.

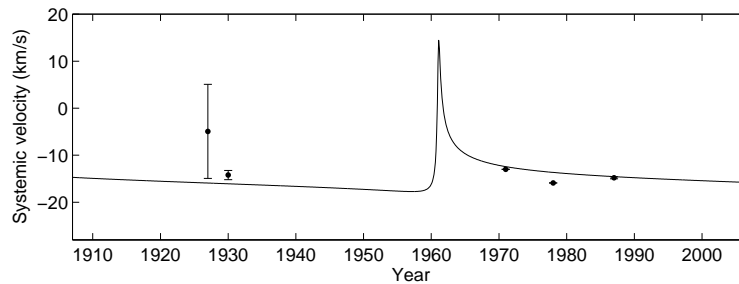


Figure 4.3: Systemic velocity variations in QS Aql. The individual points represent derived systemic velocities (see details in text), while the solid curve represents the variation on the long orbit described by the parameters in Table 4.1.

Using the derived parameters and Eq. 2.15, also the mass of the third component of the system was computed. If the total mass of the eclipsing binary was assumed to be $M_{12} = 5.9 M_{\odot}$ (according to Holmgren 1987), one obtains the third mass $M_3 = 16.5 M_{\odot}$, large primarily due to the relatively low orbital inclination of the wide orbit. This means that the third body is much more massive than the individual components of the eclipsing pair, which seems unlikely. One has to take into account the errors of the resultant parameters. Due to relatively large errors of the inclination and mass function, the resultant mass could be somewhere between 3.7 and $93.5 M_{\odot}$, which is very wide range of masses. The masses in the lower part of this interval are sufficient to get a reasonable luminosity of the third body.

Heintze et al. (1989) discussed the spectroscopic observations by Holmgren (1987) and the light-curve observations and also concluded that the third light is 1.2 times larger than the combined light of the eclipsing pair: $l_3 \approx 1.2 \cdot l_{12}$, which means that in the bolometric magnitude, the third component should be for about 0.2 magnitude brighter than the eclipsing binary: $M_{\text{bol}3} \approx M_{\text{bol}12} - 0.2$ mag. Adopting the spectral types of the primary and secondary to be B5V and F3 (i.e. $M_{\text{bol}12} \approx -2.5$ mag), the third body should have the spectral type B4. If the third star is a main-sequence object, it should have a mass of about $5.2 M_{\odot}$.

This result lies within the range of the masses received from the combined analysis. In conclusion, the presented solution is of a low accuracy, mainly due to a very incomplete

Table 4.2: The minimum times of QS Aql from photoelectric photometry.

HJD-2400000	Prim/Sec	Epoch	Ref.
23963.75	Prim	-6557.0	[1]
26159.12	Sec	-5683.5	[1]
26160.37	Prim	-5683.0	[2]
30920.55	Prim	-3789.0	[3]
37490.300	Prim	-1175.0	[4]
37799.446	Prim	-1052.0	[4]
38259.397	Prim	-869.0	[4]
38577.35	Sec	-742.5	[4]
38578.604	Prim	-742.0	[4]
38945.548	Prim	-596.0	[4]
39360.255	Prim	-431.0	[4]
40443.489	Prim	0.0	[4]
40453.544	Prim	4.0	[5]
40790.349	Prim	138.0	[5]
41182.401	Prim	294.0	[6]
44439.6649	Prim	1590.0	[7]
48501.190	Prim	3206.0	[8]

Ref.: [1] - Guthnick (1931); [2] - Guthnick & Prager (1934); [3] - Groeneveld (1947); [4] - Knipe (1971); [5] - van der Wal et al. (1972); [6] - Knipe (1972); [7] - Skillman (1982); [8] - Perryman & ESA (1997).

coverage of the astrometric orbit, but leads to an acceptable solution within the limits of the errors.

4.2 VW Cep

The eclipsing binary VW Cep (HD 197433, BD +75 752, HIP 101750) was classified as W UMa system and in fact it is one of the most often observed and analyzed system. Its magnitude is about 7.3 in V filter, but its spectrum is problematic to classify. Popper (1948) and also Kaszas et al. (1998) classified the system as K1+G5, while Pribulla et al. (2000) proposed the spectral types G5V + G8V, but Kaszas et al. (1998) noted that the spectral type G5 is inapplicable and Hill (1989) presented the spectral type K0V. Both components are chromospherically active. VW Cep is rather atypical, because during the primary (the deeper one) eclipse is the less massive star (the hotter one) occulted by the larger companion (the more massive and the cooler one).

The first observations of its light variations were done by Schilt (1926). Since 1946, a large amount of photoelectric observations was obtained. However, the observed minima times did not fit the ephemeris due to the LITE and the mass transfer between components. There were many light-time effect studies of this system and Herczeg & Schmidt (1960) proposed the presence of a third body with an orbital period of 29 years and an angular distance of the third component between $0.5''$ and $1.2''$.

In 1974, the first successful visual observation of the third component was obtained and since then, there were 16 observations of it. Regrettably, the observations near the periastron passage are missing (the gap in data is from 1991 to 1999). The last two measurements ($\theta = 231.4^\circ$, $\rho = 0.702''$ and $\theta = 232.6^\circ$, $\rho = 0.695''$) were obtained and kindly sent by Elliot Horch by a speckle camera in April 2007 (priv.comm.).

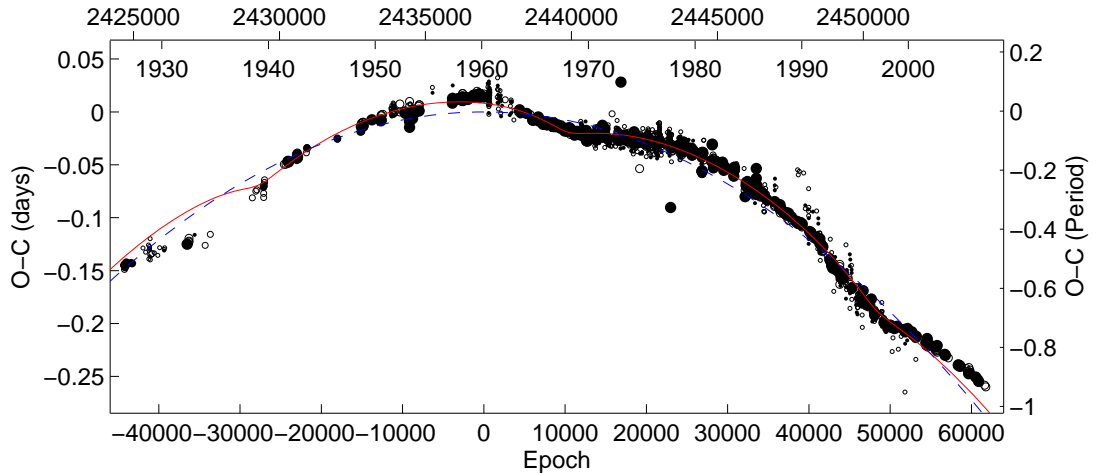


Figure 4.4: An $O - C$ diagram of VW Cep using *Solution I*. The description is the same as in Fig. 4.1. The bigger symbols are CCD and photoelectric, while the smaller ones are visual. Most of the recent visual observations were neglected. The blue dashed line represents the quadratic term and the red solid line the quadratic plus the LITE caused by the third body.

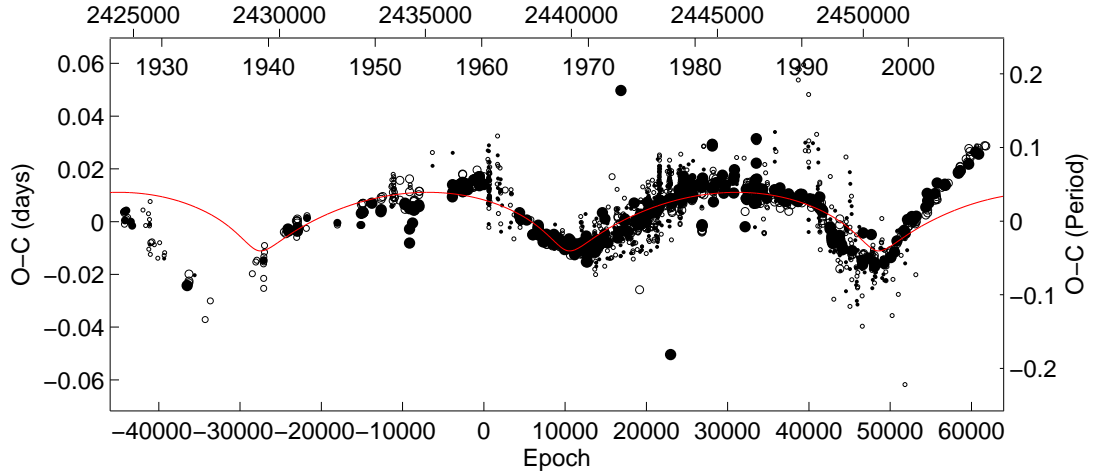


Figure 4.5: An $O - C$ diagram of VW Cep after the subtraction of the quadratic term, using *Solution I*. The description is the same as in the previous $O - C$ figures, and the red solid line represents the LITE caused by the third component in the system.

The most complete set of times of minima is in the most recent period study of VW Cep by Pribulla et al. (2000). The first times of minima are from the 1920's and altogether 1907 minima were collected. From this set of times of minima 313 measurements were neglected due to their large scatter (mostly the visual ones). This new minimum-time analysis is based on a larger data set (about 750 times of minima more than were used by Pribulla et al.), see Fig.4.4. Two new CCD observations of minimum light of VW Cep were obtained at Ondřejov observatory.

The short-term variations with the period of about two years (see e.g. Kwee (1966) and Hendry & Mochnacki (2000)) are probably caused by the surface activity cycles on the primary component. Due to this activity an unique interpretation of the behaviour of period changes is still missing. Pribulla et al. proposed a mass transfer (the quadratic term) plus the third and the fourth body in the system (two periodic terms). Nevertheless, they were not able to explain the $O - C$ diagram in detail.

Another approach was chosen in this thesis. Especially due to only a few astrometric observations (16 measurements from 1974 to 2007) it was decided to explain only the most significant effects in the $O - C$ diagram. There were two different approaches used. In *Solution I*, only the third-body orbit besides the mass transfer (the quadratic term) was considered, while in *Solution II*, instead of mass transfer the fourth body on its very long orbit and the third body was considered. This approach was chosen especially because of the systemic-velocity variations, see below. The astrometric variation with a period of about 30 years has been identified with the $O - C$ variation with the same period.

4.2.1 Solution I.

The analysis of the times of minima together with the astrometry led to the parameters shown in Table 4.3 and the $O - C$ diagram in Fig. 4.4. The times of minima, together with the curve which represents the LITE and the mass transfer, are shown in this figure.

Table 4.3: The final results: VW Cep, Solution I. and II. The table is divided into three parts, in the first one are eleven computed parameters, in the second one the values from the literature and in the last one the quantities computed from the previous parts. The values of parallax and distance were adopted from the *Hipparcos* measurements.

Parameter	Unit	VW Cep – Solution I.	VW Cep – Solution II.
JD_0	[HJD]	2437001.5289 ± 0.0034	2437001.4362 ± 0.0025
P	[day]	$0.278316241 \pm 0.00000011$	$0.278315234 \pm 0.00000012$
q	[day]	$(0.756 \pm 0.032) \cdot 10^{-10}$	–
p_3	[yr]	30.04 ± 0.47	29.99 ± 0.34
T_0	[HJD]	2450402 ± 91	2450366 ± 52
ω	[deg]	235.58 ± 3.01	242.53 ± 2.89
e		0.628 ± 0.035	0.610 ± 0.014
A	[day]	0.0117 ± 0.0009	0.0119 ± 0.0010
a	[mas]	447.4 ± 24.3	451.4 ± 28.1
i	[deg]	30.2 ± 4.2	28.0 ± 3.1
Ω	[deg]	200.0 ± 7.1	192.1 ± 5.7
M_{12}	[M_\odot]	1.37	1.37
References		Kaszas et al. (1998)	Kaszas et al. (1998)
π	[mas]	36.16 ± 0.97	36.16 ± 0.97
D	[pc]	27.7 ± 0.7	27.7 ± 0.7
a_{12}	[AU]	4.30 ± 0.41	4.59 ± 0.45
$f(M_3)$	[M_\odot]	0.0112 ± 0.0078	0.0111 ± 0.0068
M_3	[M_\odot]	0.73 ± 0.32	0.80 ± 0.30

After subtraction of the quadratic term, one gets Fig. 4.5, where only the LITE caused by the third component is displayed. The quadratic term quotient $q = -0.756 \cdot 10^{-10}$ day leads to the period change of about $1.98 \cdot 10^{-7}$ day/yr and the rate of mass transfer from the primary component of about $1.30 \cdot 10^{-7} M_\odot/\text{yr}$ (while Pribulla et al. (2000) derived the value $1.38 \cdot 10^{-7} M_\odot/\text{yr}$).

The fit is not very satisfactory because of the presence of the chromospheric activity of the individual components, or due to the putative fourth component (see e.g. Pribulla et al. 2000). It is evident, that the recent times-of-minima observations deviates from the predicted fit. This could be caused by a period jump near 1995. In Fig. 4.6, the astrometric orbit of the binary with the individual measurements and their theoretical positions is shown. Regrettably, no observations near the periastron passage are available. The curve represents the theoretical orbit according to the parameters given in Table 4.3 in agreement with the LITE analysis. Also the orbit according to the *Solution II.* is shown, see below.

The parameters describing the LITE and astrometric variation are in Table 4.3 and could be compared to the parameters derived during the previous analysis by Pribulla et al. (2000). Their values for the third-body orbit are: $p_3 = 31.4$ yr, $e = 0.77$, $\omega = 183^\circ$, and $a_{\text{total}} = 12.53$ AU. Our values are in Table 4.3 except for $a_{\text{total}} = 12.35$ AU, and as we can see they differ significantly in several parameters. This is due to completely different approach describing the $O - C$ variations. Only the period and the amplitude of such variation are comparable, but these are the most important for our combined solution.

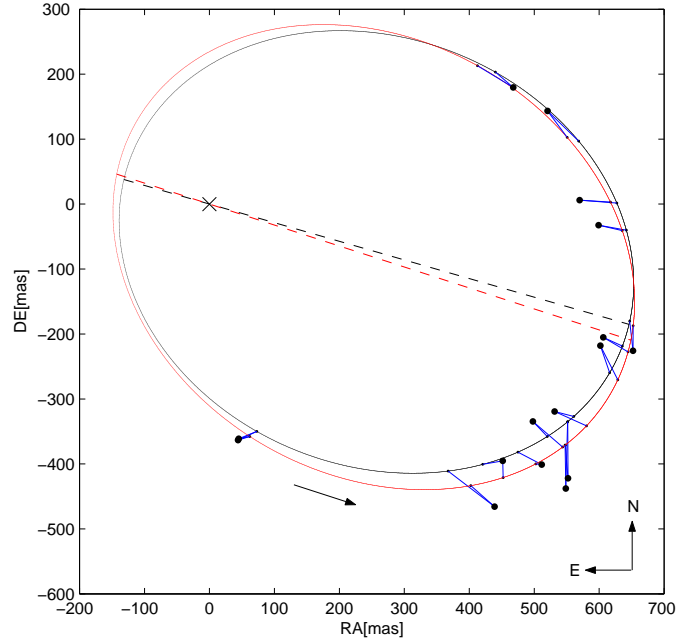


Figure 4.6: Relative orbit of VW Cep on a plane of the sky, for a detailed description see Fig. 4.2. Both *Solution I.* and *II.* are plotted, the black one for *Solution I.* and the red one for the *Solution II.*

One has also to disagree with the result by Pribulla et al., that the astrometric orbit could not be identified with the LITE₃ variation from the $O - C$ diagram. As one can see, our new results are in agreement with each other without any problems.

Also the astrometric orbit could be compared with the previously published one. Most recently Docobo & Ling (2005) published the following parameters of the astrometric orbit: $p_3 = 31.0$ yr, $a = 485$ mas, $i = 39.3^\circ$, and $e = 0.68$. If one compares these values with the new ones (see Table 4.3), one can see that the differences are slightly beyond the limits of errors.

If the total mass of the eclipsing binary $M_{12} = 1.37 M_\odot$ was taken from (Kaszas et al. 1998) and the parallax $\pi = 36.16$ mas (from Perryman & ESA 1997), the distance to the system should be only about 27.66 pc, which results in the third-body mass of $M_3 = 0.73 M_\odot$. The distant component is about 2.2 magnitudes fainter than the VW Cep itself, so its luminosity and also mass should be much smaller than the mass of the eclipsing components. Total bolometric magnitude of VW Cep is about 4.7 mag, so the magnitude of the third component is about 6.9 mag, which leads to the spectral type of about K3. The typical mass of this spectral type is about $0.75 M_\odot$ (according to Harmanec 1988), which is in an excellent agreement with our result and within its error limits.

Different systemic velocities v_γ were found at different epochs. These values are: $v_\gamma = (-35.4 \pm 10) \text{ km} \cdot \text{s}^{-1}$ (Popper 1948), $(+9.8 \pm 7) \text{ km} \cdot \text{s}^{-1}$ (Binnendijk 1966), $(-8 \pm 1) \text{ km} \cdot \text{s}^{-1}$ (Hill 1989), and $(-16.4 \pm 1) \text{ km} \cdot \text{s}^{-1}$ (Kaszas et al. 1998). In the time plot (see Fig. 4.7) one can see the curve which represents the theoretical variation of v_γ caused by the orbital motion around the common barycentre. Except for the first

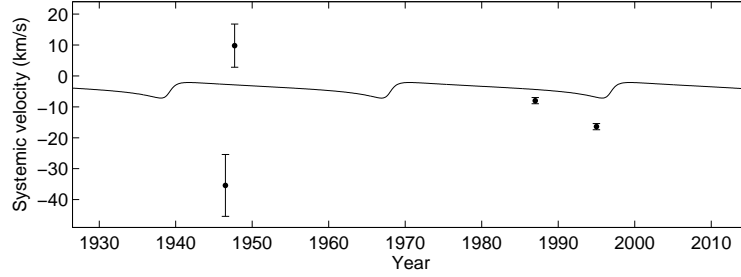


Figure 4.7: Systemic velocity variations in VW Cep, using *Solution I*. The individual points represent computed systemic velocities (see details in text).

one data point (Popper 1948), the amplitude of the LITE should be much larger (circa 3 times) than was computed. Keeping the astrometric amplitude at the value from the fit, this could only be achieved by decreasing the inclination to smaller values and by modifying slightly also some other parameters. This speculation could only be verified after more accurate and larger data set is available. But considering the spectral analysis and efficiency of the RV investigations, this result is not very satisfactory, which is the reason why the different approach was also used, see *Solution II*. below, which describes better the systemic velocity variations.

The star was also measured by the *Hipparcos* satellite. During its 3-yrs mission there were altogether 68 observations obtained, see Fig. 4.8. These are so-called *abscissa measurements* and are one-dimensional. It means that only a time of passage through a certain main circle was measured, but one cannot derive exactly where on this circle the star really was. The position of these circles (measurements) are represented by the small abscissae in Fig. 4.8. The observations are connected by a dotted lines with the theoretical positions on the sky marked as big points. The theoretical orbit was constructed according to the parameters from Table 4.3. As one can see, regrettably only a small part of the orbit was measured, so the *Hipparcos* abscissae measurements are not very useful at all.

Another task was to derive the parallax of VW Cep using this combined approach. Leaving the parallax as another free parameter, one is able to calculate it from the comparison of the angular and absolute semimajor axis (see section 2.6). Using this method, most of the relevant parameters remained nearly the same as above, only the inclination changed a bit, being about 5° lower. This led to a higher third mass of $M_3 = 0.99 M_\odot$. The main difference was in the parallax, which decreased from 36.16 mas (*Hipparcos*) to 33.63 mas. The parallax would shift the distance to 29.74 pc. New value is only about 2 pc higher then the value derived from the *Hipparcos* measurements. With more precise data points and better coverage of the orbit also this result would be better. For a comparison with the previously found parallaxes, see the next section.

4.2.2 Solution II.

From the analysis of times of minima and using the long period perturbation by another distant component instead of the quadratic term, one gets Fig. 4.9 and after the subtraction of the fourth component Fig. 4.10. It means during the computation process

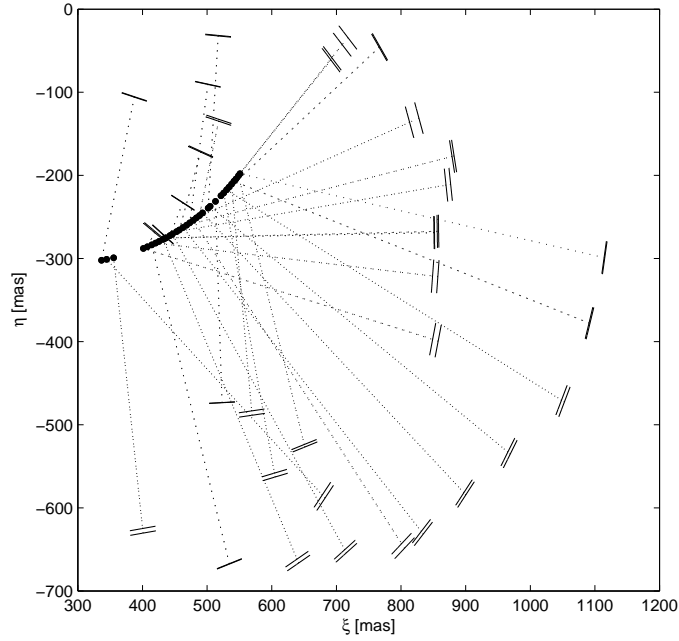


Figure 4.8: The Hipparcos measurements of VW Cep, see the text for details.

altogether 14 parameters ($A, p_3, i, e, \omega, \Omega, T_0, JD_0, P, A_4, p_4, e_4, \omega_4, T_{0,4}$) were derived minimizing the χ^2_{comb} value.

Applying this approach to the same data one gets about 12 % better result (comparing the sum of square residuals). The numerical values for the individual parameters are approximately the same (see Table 4.3). The parameters of the fourth-body orbit are in Table 4.4, where $M_{4,min}$ denotes for the minimal mass of the fourth body ($i_4 = 90^\circ$). It is obvious that the period p_4 is about as long as our data set. One can judge that this numerical solution is only an edge-on effect, which fits better the most recent data points. As one can see, the times of minima since 1995 deviate from the theoretical prediction and also an additional period jump should be implemented into the model to describe the data points in detail. Nevertheless, this combined approach was chosen because of the RV data (see below). In next few years the behaviour of the times of minima will decide which solution is the right one. Until that time this approach is just a hypothesis without any proof, only for a better description of the systemic-velocity variations.

The parameters of the third-body orbit according to *Solution II.* are close to the values from *Solution I.*, that the theoretical orbit of the binary on the plane of the sky is almost the same, see Fig.4.6.

Table 4.4: The parameters of the fourth-body orbit, VW Cep Solution II.

Parameter	p_4	$T_{0,4}$	ω_4	e_4	A_4	$f(M_4)$	$M_{4,min}$
Unit	[yr]	[HJD]	[deg]		[day]	[M_\odot]	[M_\odot]
Value	77.32	2397303	282.3	0.561	0.096	0.795	2.64
Error	± 0.04	± 14	± 2.3	± 0.008	± 0.012	± 0.055	± 0.45

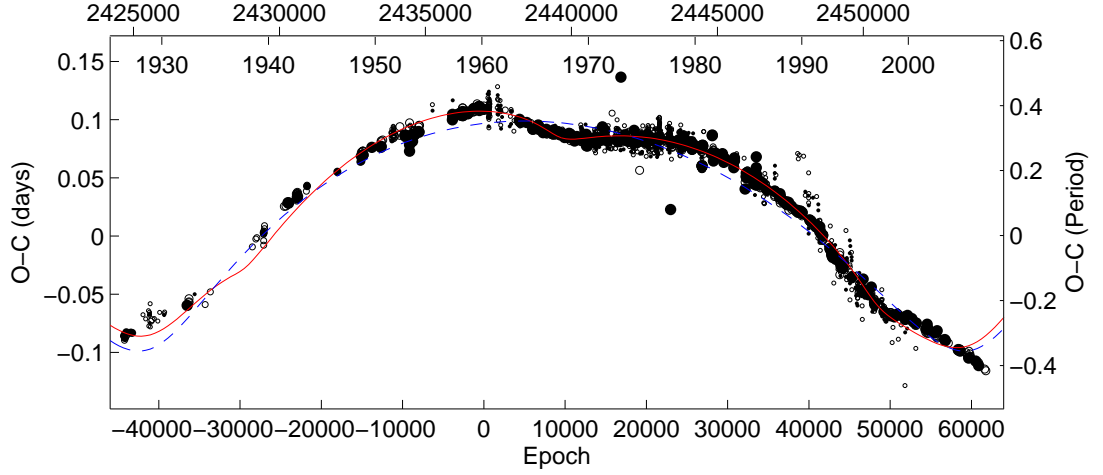


Figure 4.9: An $O - C$ diagram of VW Cep using *Solution II*. The description is the same as in Fig. 4.4. The blue dashed line represents the LITE caused by the fourth distant body and the red solid line the final fit $\text{LITE}_3 + \text{LITE}_4$.

The only effect which is significantly different, are the gamma-velocity variations. It is shown in Fig. 4.11, where the dashed and the solid line represent the LITE_4 and $\text{LITE}_3 + \text{LITE}_4$ variations, respectively. As one can see, this approach gives much better fit. Except for the first one data point (Popper 1948) the systemic velocities are following the long-term variation and are almost within its errors near the theoretical values. The value by Popper is affected by relatively large error. The scatter of the individual RV data points by Popper is larger than those from Binnendijk, which could be caused by the combination of two different data sets from different instruments and obtained after more than 600 orbital revolutions (which could shift the ephemeris). Pribulla & Rucinski (2006) suggested that the scatter of the systemic velocity data points is instrumental,

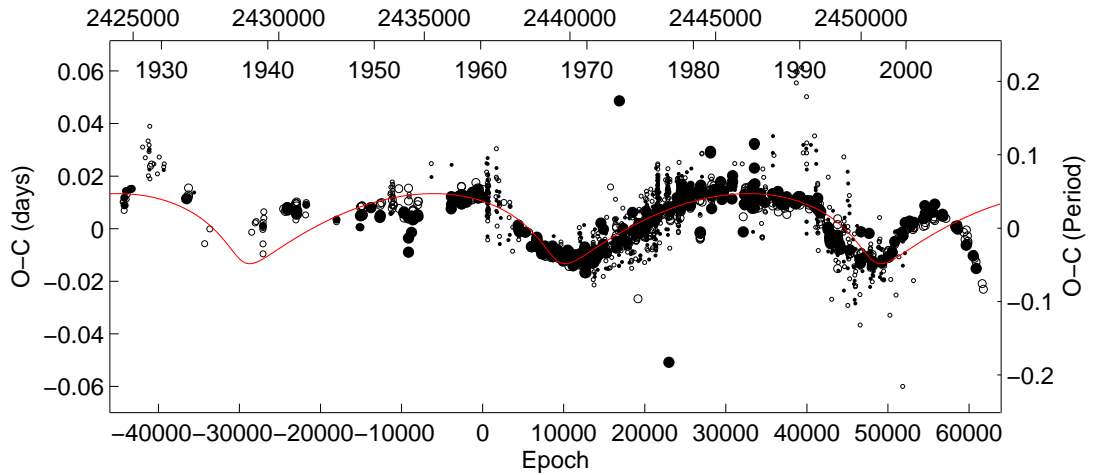


Figure 4.10: An $O - C$ diagram of VW Cep using *Solution II*, after the subtraction of the LITE caused by the distant fourth body. The description is the same as in Fig. 4.5.

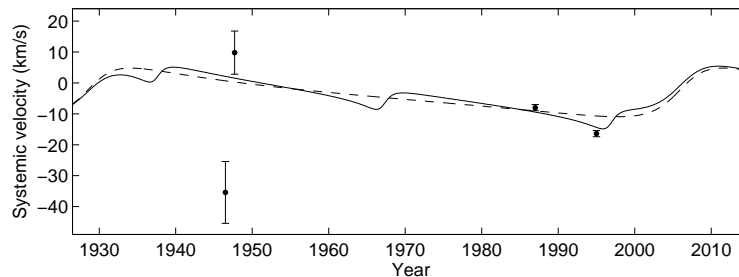


Figure 4.11: Systemic velocity variations in VW Cep, using *Solution II*. See details in text.

which seems unlikely for such a large amplitude. For the final confirmation of v_γ variations more accurate and larger data set is necessary.

One could also speculate about the possible visual detection of the fourth component. This suggested body is bright enough to be visible and its predicted angular separation from the system is about $1.1''$. On the other hand one has to take into consideration that the period of such a body is not completely covered by data and could be even higher, as well as the amplitude could be much higher (and also the angular separation). Nowadays there is no potential star for this, only the star BD+74 889 shares common proper motion and radial velocity, but it is one degree distant.

Another task was the distance determination. Due to only slight difference between the parameters of the third-body orbit from *Solution I.* and *II.*, also the parallax and distance will be approximately the same. The parallax decreased from (36.16 ± 0.97) mas (*Hipparcos*) to (35.85 ± 0.37) mas. This parallax would shift the distance from (27.7 ± 0.7) pc (*Hipparcos*) to (27.90 ± 0.29) pc. Besides the *Hipparcos* value, the most precise parallax was derived by Heintz (1993) from trigonometry, resulting in (38.2 ± 1.9) mas. As one can see, the result of the parallax is more precise than any of the previously derived parallaxes. For the summary of the previously derived values see Hendry & Mochnacki (2000).

To conclude, the predicted third body is spectral type K3 with the mass around $0.73 M_\odot$ (according to the *Solution I.*), or spectral type K2 with the mass around $0.80 M_\odot$ (applying the *Solution II.*). It is clear, however, that a more complicated model will be needed to describe the observed changes completely. Also new times-of-minima observations would be helpful to identify the variations in $O - C$ diagram in detail, because neither the *Solution I.*, nor the *Solution II.* are able to describe the behaviour of the recent minima observations. This could be described only applying the hypothesis of an abrupt period jump. Precise RV investigation (till 2010) would solve the question about the nature of the variations in gamma velocity.

4.3 ζ Phe

The system ζ Phe is the brightest eclipsing binary with two components of early spectral types, exhibiting total and annular eclipses. This is the only binary with an eccentric orbit included in this study. ζ Phe (HD 6882, HR 338, HIP 5348) is an Algol-type eclipsing binary. Apparent brightness of the system is about 4.0 mag in V filter and the spectral

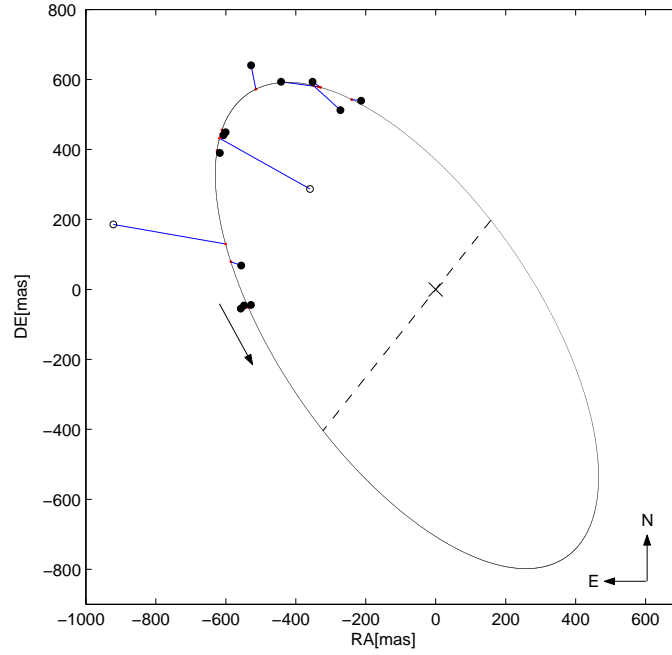


Figure 4.12: Relative orbit of ζ Phe on a plane of the sky, for a detailed description see Fig. 4.2. Two measurements (the open circles) were neglected.

types were determined as B6V + B8V (according to Andersen (1983), see also a comment on the spectral types and *Hipparcos* measurements in Ling 2004). It is a visual triple and SB2 spectroscopic binary. The depth of the primary minimum of the eclipsing pair is about 0.5 mag, the period of about 1.7 day.

It is the visual triple system, while the brightest component is the EB, the most distant component is the faintest (some $6''$ away and with a magnitude of about 8, this star is probably not gravitationally bounded with the system). The third component is a 7th-magnitude star at a distance of about 600 mas. This is the astrometric component and this star is supposed to cause also the LITE variation.

The first astrometric observation of the third component came from 1930's and till now there were collected 14 observations, but two of them were neglected (see Fig. 4.12).

The unfiltered light curve was observed in 1950's by Hogg (1951), after then by Dachs (1971) in *UBV* filters, and the best one by Clausen et al. (1976) in *ubvy* filters. In this latter paper all relevant parameters of the eclipsing system were derived and also the third light was computed. Its value changes from 3%(*u*) to 8%(*y*) and the distant component was classified as a spectral type A7 star (the same result was derived by Andersen (1983) on the basis of his spectroscopic observations).

Clausen et al. (1976) also collected the times of minima obtained before 1975. They concluded that no significant apsidal motion is observed. The first apsidal-motion study was published by Gimenez et al. (1986). With an updated list of the times of minima one is able to conclude that the apsidal motion is definitely presented. It is clearly visible in the *O – C* diagrams shown in Figs. 4.13 and 4.14. Altogether 36 times of minima used here came from the paper cited above and from Mallama (1981), Gimenez et al. (1986),

Table 4.5: The parameters of ζ Phe.

Parameter	JD_0	P	p_3	A	T_0	ω	e	i	Ω	$f(M_3)$	M_3
Unit	[day]	[day]	[yr]	[day]	[day]	[deg]	-	[deg]	[deg]	$[M_\odot]$	$[M_\odot]$
Value	2441643.7382	1.6697772	220.9	0.0808	2419900	97.1	0.366	64.4	33.5	0.056	1.73
Error	0.0008	0.0000013	3.5	0.0080	2500	2.2	0.082	3.0	4.9	0.017	0.26

Kvív et al. (1999). The most recent ones are taken from Zasche & Wolf (2007) (in print).

ζ Phe has one of the shortest apsidal motions among the eclipsing binaries (see e.g. Claret & Gimenez 1993). Due to a low eccentricity, the amplitude of the effect is small. For an accurate calculation of the apsidal motion rate the method described by Gimenez & Garcia-Pelayo (1983) was routinely used. The eccentricity of the orbit in the eclipsing binary is $e' = 0.0107 \pm 0.0020$, the longitude of periastron $\omega_0 = 12.96^\circ \pm 5.96^\circ$, and the apsidal motion rate $\dot{\omega} = (0.028 \pm 0.001)^\circ/\text{cycle} = (6.16 \pm 0.20)^\circ/\text{yr}$, i.e. the apsidal motion period $U = 58.5$ yr. The most recent apsidal-motion analysis is more than 20 years old, made by Gimenez et al. (1986), but with no LITE and with a smaller set of times of minima. The eccentricity by Giménez was almost the same, but the apsidal motion rate $\dot{\omega}$ was $0.0373^\circ/\text{cycle}$ and the angle $\omega_0 = 13^\circ$.

The approach presented here was a combination of the two different effects. The behaviour in $O - C$ diagram was described as a sum of apsidal motion and LITE contribution $(O - C) = (O - C)_{\text{apsid}} + (O - C)_{\text{LITE}}$, distinguishing the primary and secondary minima. It means the least-squares algorithm was minimizing the χ^2_{comb} with respect to 12 parameters in total ($A, p_3, i, e, \omega, \Omega, T_0, JD_0, P, \dot{\omega}, \omega_0, e'$).

The astrometric solution based on the combined approach is satisfactory, while the older measurements have larger scatter than the recent ones (the old ones are visual and the modern speckle-interferometric). Two measurements were neglected, because of their large scatter (see Fig.4.12). The solution led to the parameters listed in Table 4.5. One could compare this new orbit with the previously found one, the most recently

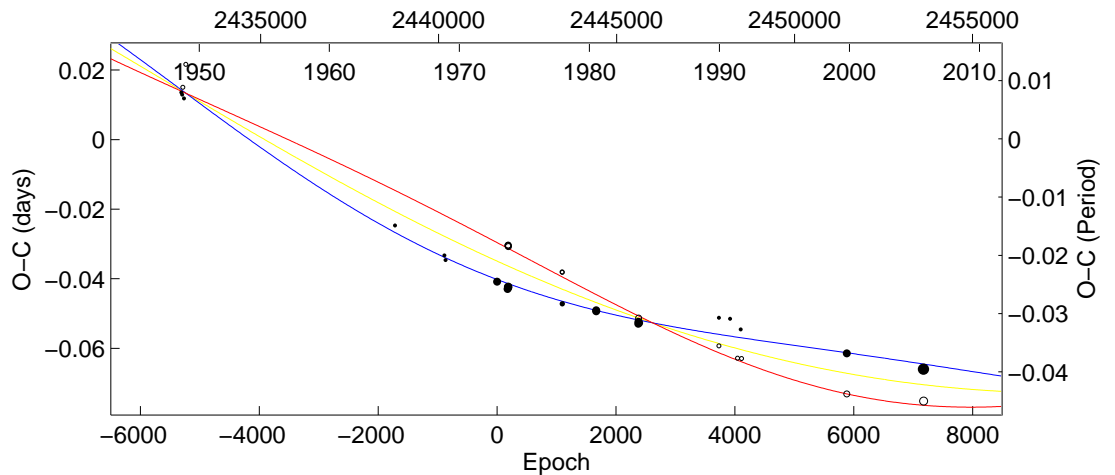


Figure 4.13: The $O - C$ diagram of ζ Phe. The apsidal motion curve (the blue one for primary and the red one for secondary) is plotted around the (yellow) LITE curve.

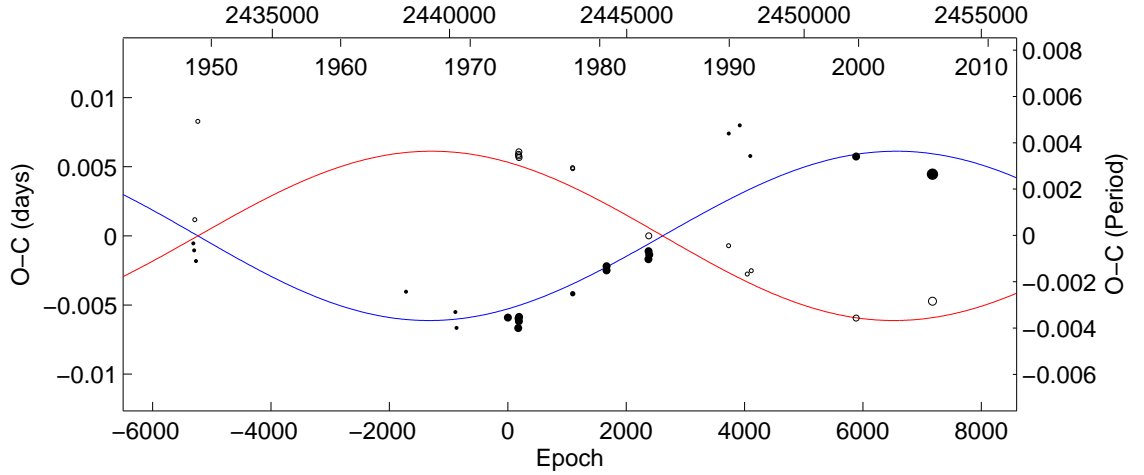


Figure 4.14: The $O - C$ diagram of ζ Phe after subtraction of the LITE. Only apsidal motion curve is shown (the blue one for primary and the red one for secondary).

Ling (2004) reported the parameters: $p_3 = 210.4$ yr, $e = 0.348$, $a = 804$ mas, $i = 61.9^\circ$, $\Omega = 33.5^\circ$, $\omega_3 = 271.7^\circ$. It is evident that the new parameters are in very good agreement with these ones. The new values imply the mass function of the distant body $f(M_3) = 0.056 M_\odot$ and with the masses of primary and secondary component of the eclipsing binary $M_1 = 3.93 M_\odot$ and $M_2 = 2.55 M_\odot$ (Andersen 1983), the mass of the astrometric third body was derived $M_3 = 1.73 M_\odot$. This value corresponds to a spectral type around A7, which is in excellent agreement with the photometric analyses by (Clausen et al. 1976) and (Andersen 1983), which result in A7.

There were also 2 RV investigations (by Popper (1970) and by Andersen 1983), but with only 2 values of the v_γ velocity one cannot do any reliable analysis. In Andersen (1983) is also presented that the lines of the third component are also observable in the spectrum of ζ Phe, but these lines are hardly separable from the binary lines.

To conclude, ζ Phe shows the astrometric as well as the LITE variations, which are in agreement with each other. Regrettably, the period of the third-body orbit was not sufficiently covered by the data yet, only about 1/4 of the orbit is covered in both methods. Only further precise measurements would prove the third-body hypothesis with higher conclusiveness.

4.4 V505 Sgr

Another EB system with apparent changes of the orbital period is V505 Sgr, where the third body is known for more than a decade. It is the Algol-type eclipsing binary with a period of about 1.2 days. V505 Sgr (HD 187949, HR 7571, HIP 97849) was classified as A2V+G5IV spectral types (according to Chambliss et al. 1993), with magnitude of about 6.5 in V filter.

The star was discovered to be an eclipsing binary by Hoffmeister (1934b). Since then a lot of light-curve measurements and analyses were carried out (for example Lázaro et al.

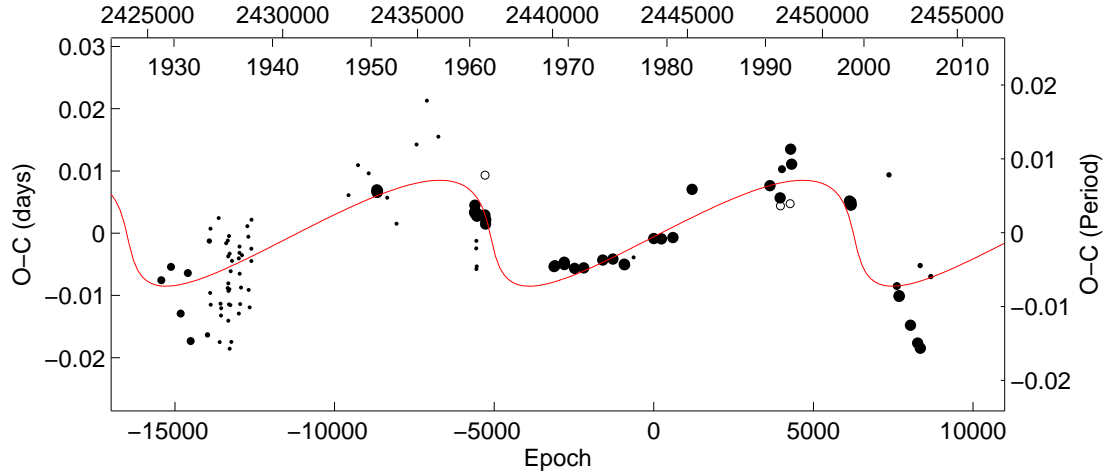


Figure 4.15: The $O - C$ diagram of V505 Sgr.

2006). The last one by İbanoğlu et al. (2000) indicated that the mass ratio of the binary is about 0.5 and the contribution to the total light of the binary by the third component is about 2.6% in B and 3.6% in V filter. This analysis also yielded the spectral type of the distant component to be roughly F6, which is in good agreement with the previous analysis by Tomkin (1992), which results in A7. The spectroscopic nature (SB2) was discovered by Popper (1949).

In 1985 an astrometric component was found by McAlister et al. (1987) by speckle interferometry. The body was $0.3''$ away from the eclipsing pair and after a few years a few measurements (16 till now) was obtained. Nowadays it is evident that the distant component is moving on its orbit around the EB pair. At the same time also Tomkin (1992) found the third component lines in the spectrum of V505 Sgr.

There are several analyses of its apparent orbital period changes interpreted as the LITE due to the third body (e.g. Rovithis-Livanou et al. 1991). The only paper which compares the astrometry and a period analysis of $O - C$ deviations from the constant orbital period was published by Mayer (1997). Despite existing astrometric measurements, there were no attempts to combine these two methods together. The results from different approaches were just compared to each other. The main reason why such a combined solution is missing, are the differences in parameters, which result from separate solutions (see below).

The $O - C$ diagram is in Fig. 4.15 and the astrometric orbit in Fig. 4.16. The last one astrometric measurement was obtained on the 18th October 2005, using 3.6-meter CFHT on Hawaii Islands, resulting in $\rho = 0.183(4)''$, $\theta = 218(2)^\circ$ (kindly sent by Theodor

Table 4.6: The parameters from the combined solution of V505 Sgr.

Parameter	JD_0	P	p_3	A	T_0	ω	e	i	Ω	$f(M_3)$	M_3
Unit	[day]	[day]	[yr]	[day]	[day]	[deg]	-	[deg]	[deg]	[M_\odot]	[M_\odot]
Value	2443750.6866	1.1828688	36.86	0.0085	2451197	184.1	0.802	195.6	22.3	0.0109	2.76
Error	0.0004	0.0000002	0.09	0.0005	19	3.2	0.008	2.4	1.9	0.0024	0.98

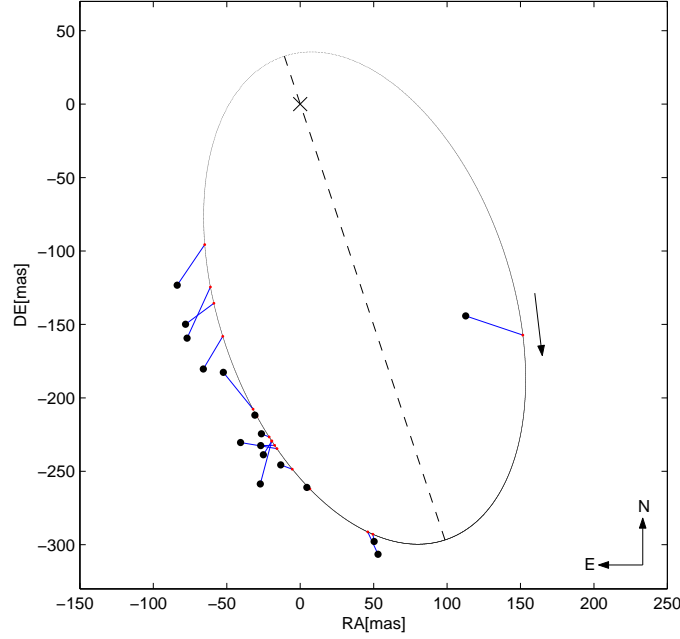


Figure 4.16: Relative orbit of V505Sgr on a plane of the sky, for a detailed description see Fig. 4.2.

Pribulla). As one can see, this point does not fit the theoretical orbit well, but one cannot ignore this data point, because it is the only measurement during the last decade and it is as precise as the previous ones (the position was obtained after averaging 5 frames).

As one can see from Fig. 4.15 also the behaviour of the last times of minima in the $O - C$ diagram is not following the theoretical curve, and it is really necessary to observe at least one precise minimum of V505 Sgr (the last one is taken from the VSNET database and is not very precise, derived only from 9 points).

The star was also measured by the *Hipparcos* satellite. Altogether 50 measurements were obtained (see Fig. 4.17). Regrettably at that time the star has not been near its periastron, so only a small arc of the orbit is covered. It is similar as in the case of VW Cep, also the description of the figure is the same.

The diagrams were plotted according to the parameters from the combined solution introduced in Table 4.6. It is obvious that the fit to the individual data points is not very satisfactory. This is due to inconsistency of the two separate solutions. Only LITE solution leads to the 41-years orbit, while the astrometric to 33yr. The angle ω differs about 60° and the amplitude of the astrometric variation is about 2 times larger than one would expect from the LITE analysis. These are the principle reasons why there is a doubt of identifying the astrometric and LITE variation to be caused by the same body.

Because the third body is also visible in the spectra of V505 Sgr, different radial velocities of the third component were measured from 1979 to 1989 (see Tomkin 1992). These measurements together with the predicted variation based on the parameters from Table 4.6 are shown in Fig. 4.18. As one can see, there is some systematic increase in the radial velocities, but due to only a small part of the period covered, this is not very conclusive result (regrettably all the measurements were obtained near the apastron).

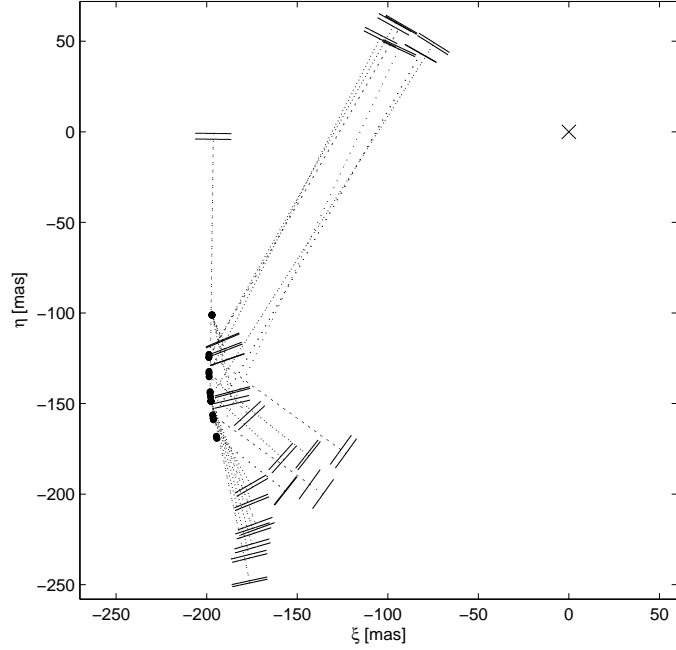


Figure 4.17: The Hipparcos measurements of V505 Sgr, see Fig.4.8 and text for details.

From the combined solution, together with the parallax from the *Hipparcos*, one is able to derive the mass of the third body. This results in $2.76 M_{\odot}$, or the spectral type of about B8 (according to Harmanec 1988). This result is in contradiction with the previous spectral analysis, which indicates a spectral type of about F6 (see e.g. Tomkin 1992), with its typical mass about $1.3 M_{\odot}$. The only acceptable explanation could be that the third component is also a binary.

In conclusion, the combined analysis of V505 Sgr leads to the results, which are not in agreement with the previous analyses. The spectral type and the mass of such a body is in contradiction with the spectral analysis by Tomkin (1992). This is especially due to the inconsistency of the results from the separate LITE and astrometry. Only one astrometric measurement was obtained during the last decade, which is not sufficient for the precise determination of the orbit. Also in the $O - C$ analysis new precise times of minima are needed to prove the LITE variation. The similar situation also apply with

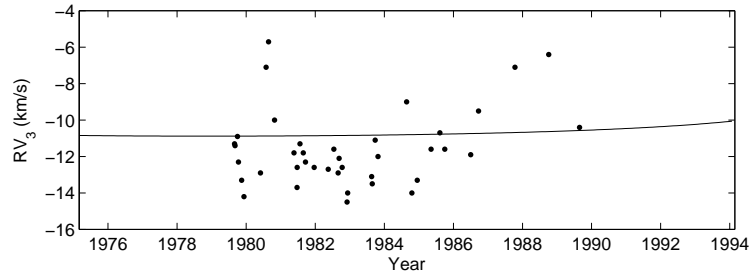


Figure 4.18: The radial velocity variations of the third component in V505 Sgr. The solid line represents the variation caused by the third body according to the parameters from Table 4.6.

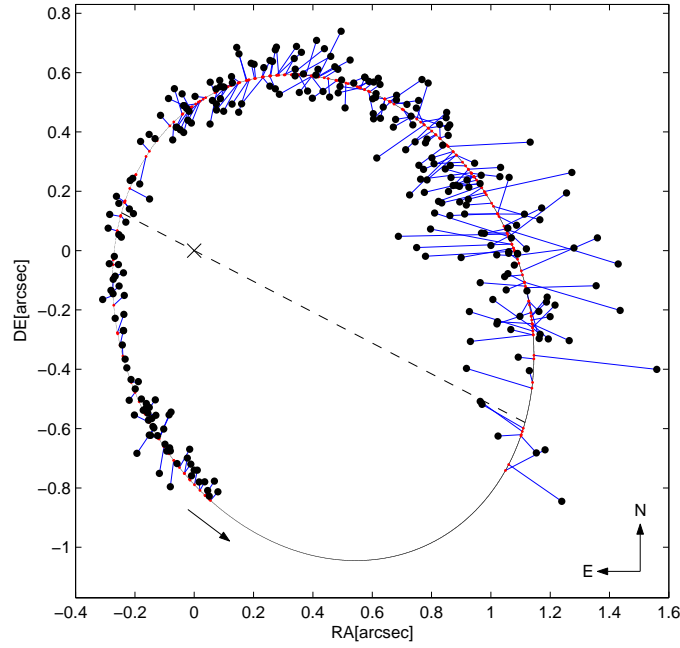


Figure 4.19: Relative orbit of HT Vir on a plane of the sky, for a detailed description see Fig.4.2.

the radial-velocity measurements. Obtaining the spectra and the radial velocity of the third component would be very helpful.

4.5 HT Vir

One member of the visual binary STF 1781 is the eclipsing binary system HT Vir (ADS 9019, HD 119931, HIP 67186, BD+05 2794). HT Vir is a contact W UMa system, with a period of about 0.4 days and the depths of minima of about 0.4 mag. Both visual components have almost equal brightness. The third component of the system is brighter than the eclipsing binary HT Vir during its eclipses and fainter than it during its maxima. The system is apparently about 7.2 mag bright in *V* filter and the spectral type was classified as F8V (according to Lu et al. 2001).

According to Walker & Chambliss (1985) the distant astrometric component was discovered by Wilhelm Struve in 1830 at a separation of about 1.4'' and position angle 240°. Since then, numerous astrometric observations were obtained (altogether 277, from which 275 were used in our analysis) and the orbit is almost completely covered by the observations (see Fig. 4.19).

Baize (1972) suggested that the star might be variable. After then, Walker & Chambliss (1985) obtained a complete light curve of HT Vir and did the first analysis. It indicated that both components of the eclipsing pair are almost identical and in contact. The temperatures of both components are about 6000 K and the spectral type is estimated as F8V. The same (combined) spectral type was derived from the spectral analysis by Lu et al. (2001), but with a strong contribution of the third component. The total mass of the eclipsing pair is $M_{12} = 2.3 M_{\odot}$ (D'Angelo et al. 2006).

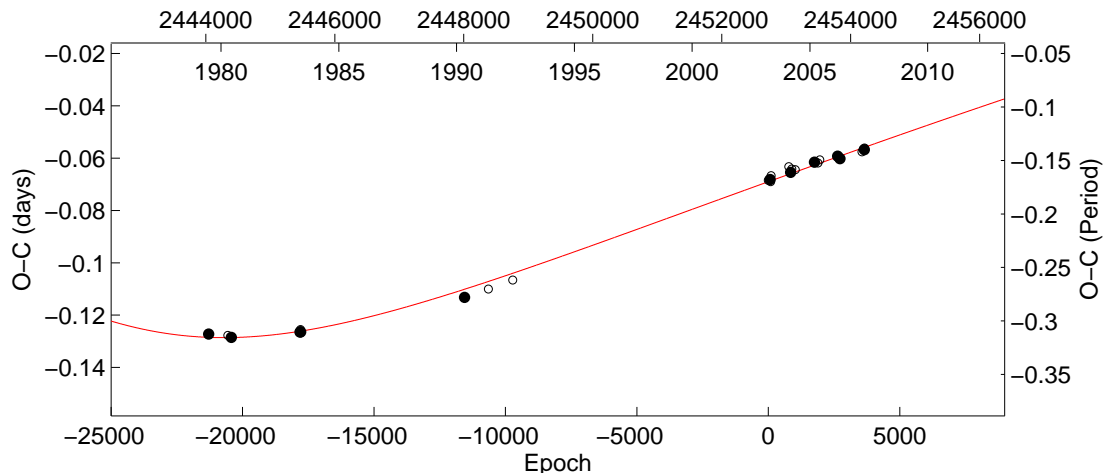


Figure 4.20: An $O - C$ diagram of HT Vir. The description is the same as in the previous $O - C$ figures, all minimum times are the photoelectric or CCD ones.

Lu et al. (2001) discovered that the distant component is also a binary. They have measured the spectra of HT Vir eclipsing pair, and discovered also the lines from the third component in the spectra and their RV variations with a period of about 32.45 days. We therefore deal with a quadruple system.

Despite the spectral analysis and a large set of astrometric observations, there were only a few times of minima published during the last few decades. The main reason is the relatively recent discovery of the photometric variability of HT Vir. The first times of minima come from 1979. Since then, there were only 31 observations obtained (see Fig. 4.20). Four new observations were obtained, two of them were observed at Ondřejov observatory, one by L.Brát and the last one by R.Dřevěný. One unpublished observation by M.Zejda was also used and four times of minima by M.Zejda published in Zejda (2004) were recalculated, because the heliocentric correction was wrongly computed.

Walker & Chambliss (1985) published the first rough estimation of the proposed amplitude of LITE from the parameters of the astrometric orbit. Their value (0.18 day) is not too far from the present one (0.13 day).

The final plot of the relative astrometric orbit of HT Vir is in Fig. 4.19. The results, the parameters of the orbit around the common barycenter of the system, are given in Table 4.7. The values of these parameters ($A, p_3, i, e, \omega, \Omega, T_0, JD_0, P$) were obtained minimizing the χ^2_{comb} .

The new elements for the astrometric orbit could be compared to these by Heintz (1986), which are the following: $p_3 = 274.0$ yr, $e = 0.638$, $a = 1010$ mas, $i = 42.7^\circ$, $\Omega = 176.4^\circ$, $\omega_3 = 250.0^\circ$. As one can see, the period of the new orbit is a bit shorter, but the main differences in these values are the angles ω and Ω . The same fit to the astrometric data could be reached with simultaneously transformed values $\omega_3 \rightarrow \omega_3 + 180^\circ$ and $\Omega \rightarrow \Omega + 180^\circ$. This only means the interchange of the role of the two components. This result therefore indicates the incorrect identification of the variable HT Vir in the system in our analysis (the variable was supposed to be the component A) and also in the

Table 4.7: The parameters from the combined solution of HT Vir.

Parameter	JD_0	P	p_3	A	T_0	ω	e	i	Ω	$f(M_3)$	M_3
Unit	[day]	[day]	[yr]	[day]	[day]	[deg]	-	[deg]	[deg]	[M_\odot]	[M_\odot]
Value	2452722.5040	0.4076696	260.7	0.1274	2442832	250.9	0.640	45.4	180.8	0.169	2.10
Error	0.0050	0.0000025	0.5	0.0026	61	0.7	0.005	3.7	2.6	0.009	0.11

WDS catalogue, see WDS notes ². While Pribulla & Rucinski (2006) correctly identified the variable HT Vir as a B component and A as a single-lined binary.

If one adopts these parameters to estimate the mass function of the distant pair (mass function of the whole pair, not the individual components), one obtains $f(M_3) = 0.17 M_\odot$. This is quite a high value, dictated by the large amplitude of the LITE. With the total mass of the primary and secondary $M_{12} = 2.3 M_\odot$ one gets the third mass of $M_3 = 2.14 M_\odot$. The mass of the distant pair is quite high (D’Angelo et al. (2006) derived the mass for some 50% lower, $M_3 = 1.15 M_\odot$), but note that also this object is a binary and we do not know the individual masses. From the spectroscopic observations (to remind, it is a SB1-type binary), one is only able to estimate the mass function of the components, or some upper limit for one of them (we do not know the inclination). The present result M_3 is the total mass of the SB1 pair $M_{3,1} + M_{3,2}$; the limit for the invisible-component mass $M_{3,2} \cdot \sin(i') = 0.075 M_\odot$. If the the coplanar orbit is assumed, high difference in masses would arise, one component should be much more luminous and also more luminous than the eclipsing pair itself, which is not the case. In fact the whole system is not coplanar (see e.g. $i = 315.5^\circ$ and the inclination of the EB close to 90°). If one assumes two approximately equal masses, there is a problem with the luminosity, because the distant pair has to be roughly as luminous as the eclipsing pair. This could only be satisfied if one component is underluminous or degenerate.

One has to take into consideration also the comment on the light-curve solution by Walker & Chambliss (1985). Using the Wood’s model, they discovered that if the third light L_3 is fixed to be the equal to the light from the distant visual component (it means $L_3 = 0.5$), the solution of the light curve is unrealistic. To conclude, the system could be much more complicated than the approach that was used here. There may be some additional component(s) or the distant pair is composed from evolved stars, away from the main sequence. Especially because of the resultant mass and luminosity of the distant pair, the body causing the astrometric variation is probably different from the one causing LITE, but this conclusion will be proven only if also the nonlinear part of the $O - C$ diagram is covered.

4.6 The problematic case: V2388 Oph

Another system, where the astrometric orbit is known and also the set of times of minima is available is V2388 Oph. The contact eclipsing binary system V2388 Oph (FIN 381, HD 163151, HR 6676) is β Lyrae type (according to Simbad database), or more likely W UMa type (according to Rodríguez et al. 1998). Its orbital period is about 0.8 days,

²http://ad.usno.navy.mil/wds/wdsnewnotes_main.txt

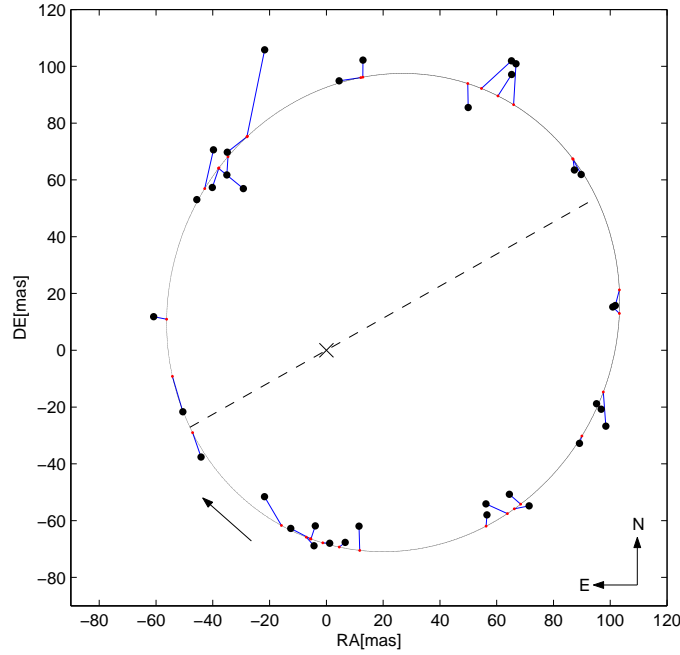


Figure 4.21: Relative orbit of V2388 Oph on a plane of the sky, *Solution I*. Three measurements were neglected due to their large scatter.

apparent brightness about 6.3 mag in V filter and the spectral type was classified as F5Vn (according to Hipparcos Catalogue, Turon et al. 1993), or F3V according to Rucinski et al. (2002). The depths of its minima are 0.3 and 0.25 mag for primary and secondary, respectively.

The first astrometric observation of the third component was obtained in 1959 (Finsen 1963) in a distance of about only 100 mas. During the next decades there was recognized rapid movement of this component around the primary. The preliminary orbit was calculated by Baise (1988) with the period of about 8.3 yr, semimajor axis $0.09''$ and eccentricity 0.29. Nowadays orbit is a little bit different ($p_3 = 8.9$ yr, $a = 0.09''$, $e = 0.33$). It is evident, that the movement is very rapid and since its discovery the body revolved 5 times around the eclipsing pair.

The photometric variability was discovered by Rodríguez et al. (1998), but the variability is also evident from the Hipparcos observations. The $ubvy$ light curves were obtained and analyzed. With the RV analysis made by Rucinski et al. (2002) one is able to get the complete picture of the system. The mass ratio from the photometry is 0.27, but more precisely from spectroscopic analysis $q = 0.186$, the minimum mass $(M_1 + M_2) \sin^3 i = 1.93 M_\odot$. According to $ubvy$ analysis the system was classified as F5Vn, but Rucinski et al. on the basis of their spectroscopic observations suggested slightly earlier spectral type F3V. The magnitude difference between astrometric components is $\Delta m = 1.80$ mag and the contribution of the third component to total luminosity is circa 20%. Mass of the third component is $M_3 = 1.36 M_\odot$ according to D'Angelo et al. (2006). The EB system is SB2-type (according to Rucinski et al. 2002) also with the third component visible in the spectra. The mean radial velocity of the third component

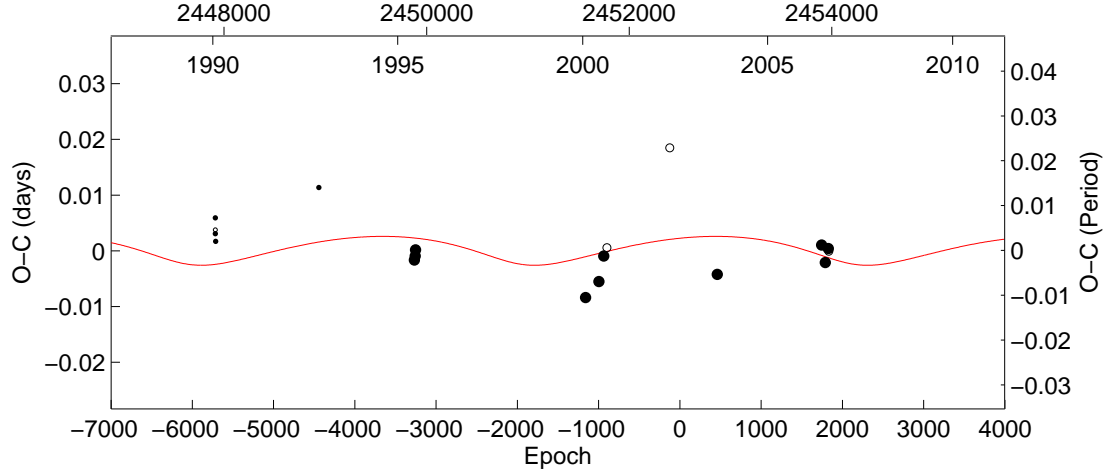


Figure 4.22: An $O - C$ diagram of V2388 Oph, *Solution I*. All minima times are the photoelectric or CCD ones (the first 5 of them are from the *Hipparcos* mission).

$V_{\gamma,3} = 30.64 \text{ km}\cdot\text{s}^{-1}$ significantly differs from the center-of-mass velocity of the binary, $V_0 = 25.88 \text{ km}\cdot\text{s}^{-1}$, which could be caused by the motion on the 9-yr orbit. In the same paper was mentioned an unexplained *cross-talk*, which means variation in radial velocities of the distant companion in phase with the period of the eclipsing binary. Having only this one RV data point one could not determine any of the relevant parameters of the long orbit.

Rough estimation of the proposed $O - C$ variation and its magnitude was done by Rucinski et al. (2002) from the astrometric orbit. Their analysis results in $A = 0.011 \text{ day}$. Since the discovery of the photometric variability of V2388 Oph the minimum light of the star was observed only 18 times. But the individual data points show different variation, with quite different amplitude, but mainly with very different period.

The astrometric orbit is plotted in Fig.4.21 and the final fit is satisfactory, with the resultant parameters given in Table 4.8 – *Solution I*. The $O - C$ diagram is in Fig.4.22. As one can see, the fit is unacceptable, but the method applied here was the same as in the previous cases.

Where the problem could be? The crucial point at the first time is to compare the results from these different approaches and take it into the consideration. If these two results are incomparable (as in this case) this combined approach is unusable. And this is the case of V2388 Oph.

Using only $O - C$ analysis to the set of times of minima, one gets the period $p_3 = 5.1 \text{ yr}$, $e = 0.56$ and $\omega = 102.4^\circ$. It is evident that the astrometry leads to the different set of parameters and the combined solution should be nonsense. Another solution could be the very short one, with the period $p_3 = 2.99 \text{ yr}$, but this is only the hypothesis, because the variation in $O - C$ diagram is not covered very well and this is only a *sampling* frequency of the individual data points.

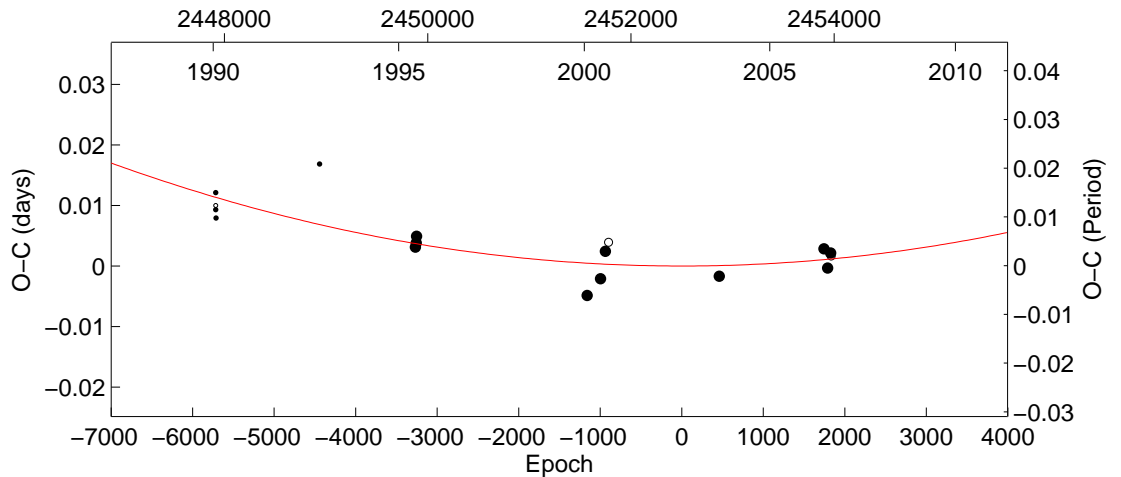
The astrometric orbit is well defined, but there is a question about the accuracy of the individual times-of-minima data points. Without the input data (the rough photometry), one can doubt, if all the measurements are accurate enough or some of them could be

Table 4.8: The final results: the case of V2388 Oph, Solution I. and II. The description is the same as in Table 4.3.

Parameter	Unit	V2388 Oph – <i>Solution I.</i>	V2388 Oph – <i>Solution II.</i>
JD_0	[HJD]	2452500.3829 ± 0.0066	2452500.3799 ± 0.0009
P	[day]	0.8022986 ± 0.0000025	0.8022995 ± 0.0000037
q	[day]	0.0	$-3.472 \cdot 10^{-10} \pm 0.010$
p_3	[yr]	9.01 ± 0.28	9.01 ± 0.29
T_0	[HJD]	2549594.9 ± 60.7	2549666 ± 77
ω	[deg]	243.6 ± 1.7	301.6 ± 1.6
e		0.329 ± 0.002	0.318 ± 0.003
A	[day]	0.0026 ± 0.0015	0.00001 ± 0.00012
a	[mas]	88.2 ± 67.4	85 ± 1080
i	[deg]	156.7 ± 2.9	180.00001 ± 1.8
Ω	[deg]	181.0 ± 4.8	240.5 ± 2.3
M_{12}	[M_\odot]	2.14	2.14
References		Yakut et al. (2004)	Yakut et al. (2004)
π	[mas]	14.72 ± 0.81	14.72 ± 0.81
D	[pc]	67.9 ± 3.7	67.9 ± 3.7
a_{12}	[AU]	1.14 ± 0.63	0.56 ± 1940
$f(M_3)$	[M_\odot]	0.0012 ± 0.0011	$0.00000002 \pm 0.00000001$
M_3	[M_\odot]	0.50 ± 0.47	0.23 ± 714.00
Data set		35a + 18m	35a + 17m

neglected. Interesting is the sequence of 4 times of minima, which are rising up near the epoch -1000 (3 primary and 1 secondary). Is this the real effect in $O - C$ diagram, or is it just the real scatter of the measurements? The difference is about 0.01 day, or circa 14 minutes. This is quite large to be only a scatter, but one does not know the conditions during the observation, etc.

If one decide to neglect one data point – the secondary minimum time near the epoch

Figure 4.23: An $O - C$ diagram of V2388 Oph, *Solution I.* – only the quadratic term was used.

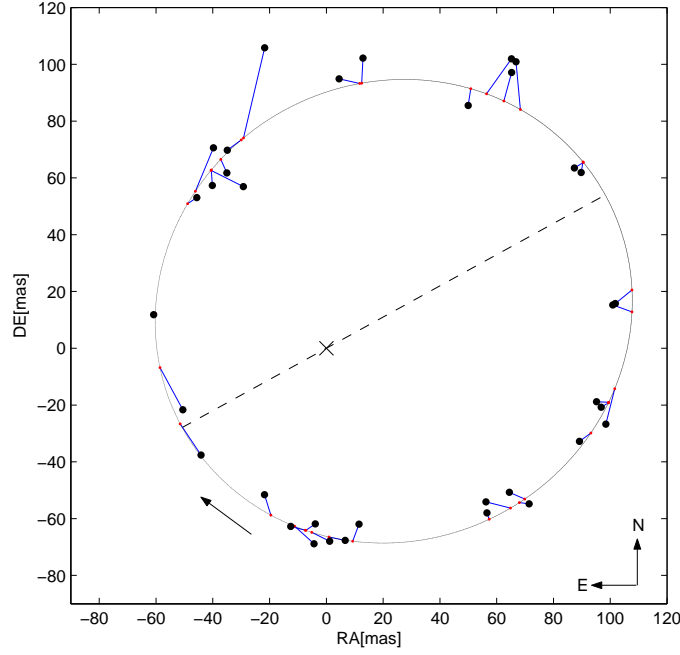


Figure 4.24: Relative orbit of V2388 Oph on a plane of the sky, *Solution II*.

0, and include also the quadratic term in the ephemeris, one will get *Solution II*. – see Fig. 4.23. Using the combined approach also the astrometric orbit could be plotted (see Fig. 4.24) and as one can see, the difference between Figs. 4.21 and 4.24 is not so significant. The main difference is in the $O - C$ diagram, where only the quadratic term arises and no LITE is presented. This means that the orbit is just *face-on*, i.e. the inclination is very close to 180° . See the resultant parameters of such fit in Table 4.8. The parameters of LITE are not very convincing, because of inclination is almost 180° , but the LITE is necessary to compute, because the amplitude of astrometric variations is computed from the amplitude of LITE. This means the mass of the third component was derived precisely, while the mass function of such a body is very inaccurate.

To conclude, it is difficult to decide, which solution is the right one. Only further data points, especially times of minima, would confirm the 9-yrs variation in the $O - C$ diagram. Due to very short period of the third body the shift in the times of minima should be evident after a few months of observations. Also measuring the RV curve and the analysis of the systemic velocity could be very helpful, because the period is short and the last one was carried out more than 6 years ago.

4.7 Another systems

In this section are presented the systems which were found to be EBs as well as members of the visual binaries. The limitation about the number of times of minima, which was presented in Introduction to Chapter 4, does not play a role. Only the systems analyzed in detail above were omitted. The systems presented here were found scanning the objects

in the WDS catalogue and trying to identify the EBs in this sample of stars. This survey is slightly following the paper on "Eclipsing binaries in multiple-star systems", Chambliss (1992). The number of such systems have grown rapidly since then, but the main difference is the selection criterion. In Chambliss (1992) are presented all of the multiple systems with eclipsing binaries, which were known for the author. Chambliss mentioned that 80 EBs are known to be components of the multiple-star systems and 37 of them were presented in more detail. For this thesis there were selected only these systems, which were discovered to be EBs and astrometric variables after Chambliss (1992), the systems which have sufficiently large data set in both methods to do the simultaneous analysis, or the systems for which the new astrometric orbit was calculated for the first time. Some of the presented systems were also included due to their misidentification as EBs.

4.7.1 HD 123

HD 123 (V640 Cas, HR 5, STF 3062AB) is an eclipsing binary which spectral type was classified as G5V. Its V magnitude is of about 5.93, but there were only a few times of minima found in the published literature, no photometric analysis was found. The eclipse observations are questionable and recent measurements indicate possible misidentifications of the star as eclipsing binary. On the other hand the astrometry covers whole orbit. Altogether 572 data points were obtained during 170 years. Söderhjelm (1999) computed the orbital parameters, the period about 107 yr and angular semimajor axis about $1.4''$.

4.7.2 HD 1082

HD 1082 (V348 And, A 1256AB, HIP 1233) is an Algol-type EB, which spectral type was classified as B9V. Its apparent magnitude is 6.76 in V filter. The same situation as in the previous case also apply here, there were neither no times of minima nor the photometric analysis found in literature. The astrometric orbit is covered by 61 data points obtained during 93 years and covering the range from 4 to 223 degrees in θ . From these data the orbit was calculated by Olević (2002), resulting in $p_3 = 138$ yr and $a = 150$ mas.

4.7.3 HD 4134

HD 4134 (V355 And, STF 52AB, HIP 3454) is also an Algol-type EB with spectral type classified as F5 and the magnitude $V = 7.69$ mag. No times of minima were obtained. Astrometry covers only 20° with 51 measurements observed in the last 170 years, the orbit was not computed.

4.7.4 HD 10543

HD 10543 (V773 Cas, BU 870AB, HR 499) is an Algol-type EB with spectral type A3V and the magnitude $V = 6.21$ mag. The orbital period of the eclipsing pair is about 1.3 days, but only one time of minimum was observed. The astrometry covers about 80°

with 79 observations made during 120 years. The astrometric orbital parameters were calculated by Popovic & Pavlovic (1995), resulting in period about 304 yr and semimajor axis about $1''$.

4.7.5 HD 12180

HD 12180 (AA Cet, ADS 1581 A, HIP 9258) is W UMa type EB, sp F2V, $V = 7.22$ mag, and orbital period about 0.54 d. There were more than 200 times of minima obtained during the last 40 years, but with no significant LITE variation. Also the astrometric observations, which were obtained during more than 200 years, do not show any evident variation and any orbital solution could be found from this data set.

4.7.6 HD 14817

HD 14817 (V559 Cas, STF 257AB, HIP 11318) is one component of the visual binary STF 257AB. It is the eclipsing binary of Algol-type, as well as spectroscopic binary, spectrum classified as B8V, apparent brightness of about 7.02 mag in V filter and orbital period of about 1.58 day. There were 7 times of minima observed since 1971 to 1991. Due to its very long orbital period, about 836 yrs (according to Hartkopf et al. 2001), only about one third of the orbit is covered by the observations (101 observations and the change in θ is about 100°). The astrometric measurements are available since 1830 and the periastron passage occurred in 1932, so the part of the orbit near periastron is sufficiently covered. Regrettably, in that time the minima times are missing.

4.7.7 HD 18925

HD 18925 (γ Per, 23 Per, HJ 2170A, HR 915) is an Algol-type EB with spectral type of about G8III and the magnitude $V = 2.95$ mag. Astrometric observations (altogether 67) were obtained during 65 years and the orbit was calculated. Pourbaix (2000) published the parameters, $p_3 = 14.6$ yr and $a = 144$ mas, and the inclination $i = 90.6^\circ$, so the orbit is just edge-on and the astrometric observations are only "in the line". The position of the orbit indicates that the occultations and eclipses may happen. These were predicted and successfully observed in September 1990 (see Griffin et al. (1994) for details). The system is similar to β Aur (see below). Therefore, this object is not suitable for the simultaneous analysis.

4.7.8 HD 19356

HD 19356 (Algol, β Per, LAB 2Aa, HR 936) is well-known prototype of the Algol-type binaries. Its spectral type is B8V and apparent brightness 2.12 mag in V . The time of minimum brightness was first measured by Montanari on 8 November 1670 (although known from historical times). Nowadays set of times of minima is really large, about 1400 observations, covers a few centuries, but the detailed description of the $O - C$ diagram is still missing. The system is rather complicated, but the distant component with the

orbital period about 1.8 yr discovered firstly on the basis of the radial velocity variations was found in 1973 by a speckle camera and the orbit of this component is now well established ($a = 94.6$ mas and $e = 0.23$, according to Pan et al. 1993).

4.7.9 HD 24071

HD 24071 (DUN 16, HR 1189, HIP 17797) is probably β Lyrae type star, its apparent brightness is about 4.2 mag in V filter and spectrum classified as B9V. The star is hardly measurable, because there are together 4 stars very close each other (only $5''$ distant) and it is not clear, if all these components belong to the system. The astrometry was obtained in 1826 for the first time, there were 80 measurements in total, which reveals the change in position angle of about 15° . The orbit computed according to these data is not very conclusive (the orbital period more than 5000 yr).

4.7.10 HD 25833

HD 25833 (AG Per, STT 71AB, HIP 19201) is an Algol-type EB spectral type B5Vp and the relative magnitude $V = 6.69$ mag. There were 101 times of minima, collected from the published literature. These minima were obtained from 1920's till now. AG Per is one of the most typical apsidal-motion systems, which has been analyzed for the apsidal motion several times (see e.g. Wolf et al. 2006). One could also apply the hypothesis of combining the apsidal motion and the LITE into one joint solution (similar to the ζ Phe case). Also the precise light curves were measured and analyzed (see Woodward & Koch 1987). The main problem arises with the astrometry. Altogether 38 measurements cover more than 30 degrees in θ , but there is no evident periodicity and the orbit could not be constructed from this data set.

4.7.11 HD 29911

HD 29911 (V592 Per, COU 1524, BD+39 1054) is a β -Lyrae EB with spectral type classified as F2 and its apparent magnitude $V = 8.37$ mag. There was only one time of minimum obtained. The astrometry covers only 19° with 18 data points obtained during

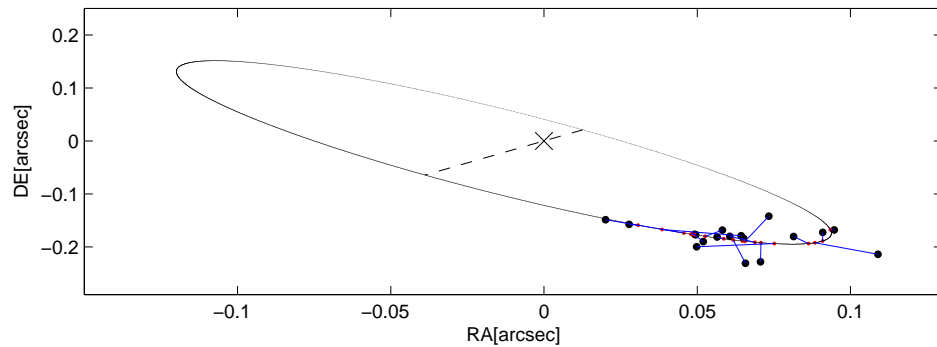


Figure 4.25: Relative orbit of V592 Per on the plane of the sky.

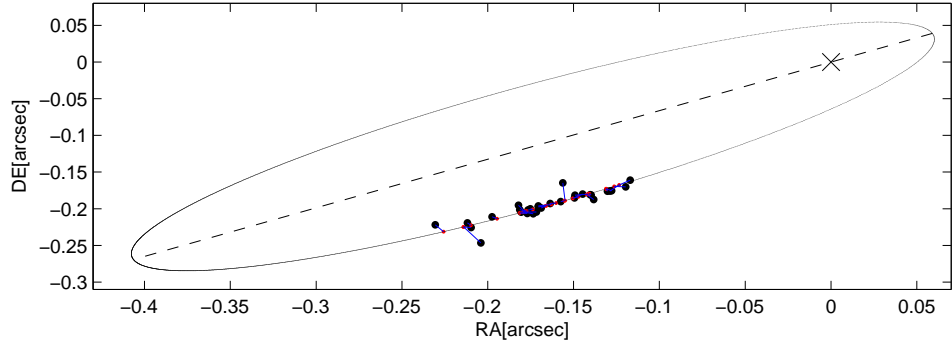


Figure 4.26: Relative orbit of δ Ori on the plane of the sky.

26 years. The plot is in Fig.4.25, where the theoretical orbit is also shown. Its period is about 117 yr and semimajor axis of about 230 mas, it was computed for the first time.

4.7.12 HD 36486

HD 36486 (δ Ori A, 34 Ori A, HEI 42Aa, HR 1852) is the eclipsing binary, sp O9.5II, $V = 2.23$ mag, and the orbital period 5.7 days. Only 9 times of minima were found in literature, but these minima do not show any significant LITE variation (more probably apsidal motion). On the other hand, there is significant motion on the plane of the sky, while the astrometry was first obtained in 1978 and since then 38 observations were obtained (see Fig.4.26). The orbit is only a preliminary one and has not been published till yet. The period of the orbit is about 313 yr and the semimajor axis 280 mas.

4.7.13 HD 38735

HD 38735 (V1031 Ori, MCA 22, HR 2001) is the Algol-type detached system, $V = 6.06$ mag, sp A4V, period about 3.41 d. There were 9 times of minima found in literature. The orbit of the binary is shown in Fig.4.27. It consists of only 20 observations obtained from 1980 to 1997. This orbit was not published yet and is only a preliminary one. Its

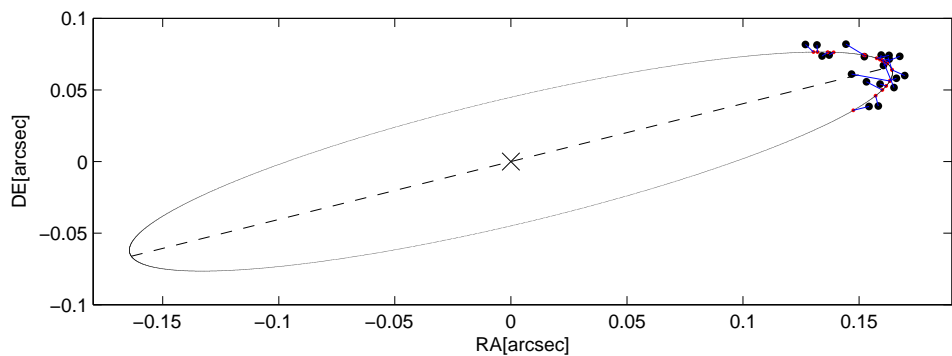


Figure 4.27: Relative orbit of V1031 Ori on the plane of the sky.

orbital period is about 92 yr and the semimajor axis about $0.18''$. But according to the RV measurements by Andersen et al. (1990), the orbit should be much larger, and the period about 3700 yr. The third-component lines were observed in the spectra of V1031 Ori and radial velocities on the 92 yr orbit would be much larger than measured. Because the orbit is covered by data only very poorly, only further astrometric observations, as well as precise radial velocity investigation will reveal the nature of the system.

4.7.14 HD 40183

HD 40183 (β Aur, 34 Aur, HR 2088) is an Algol-type EB, apparently bright about 1.9 mag in V filter and its spectrum was classified as A2IV. It is one of the brightest and nearest spectroscopic as well as eclipsing binaries, but due to its high brightness only a few observations were done. Altogether 23 times of minima were measured over the whole century. The photometric (see Johansen 1971) and also spectroscopic (see Nordström & Johansen 1994) analyses were published. Similarly to the previous case γ Per, the astrometric orbit could be identified with the eclipsing binary orbit. Therefore, this system does not belong to this survey, but it is of big importance for the present EB knowledge. Altogether 28 data points sufficiently cover the whole 4-day astrometric orbit. The EB components could be resolved, because the system is relatively close (about 24 pc). Detailed description of the technique used (interferometry with The MARK III long-baseline optical interferometer on Mount Wilson) and the analysis is in Hummel et al. (1995). The parameters of the orbit from interferometric measurements were compared by Hummel et al. with the previously found values from photometry and spectroscopy. The different approaches lead to the same results within their respective errors. Also the determination of the distance to this unique binary from four independent methods gave the comparable results.

4.7.15 HD 57061

HD 57061 (τ CMa, 30 CMa, FIN 313Aa, HR 2782) is the brightest star in the open cluster NGC 2362. It is the β Lyrae-type EB, period about 1.28 d. τ CMa is also a spectroscopic binary with the orbital period of about 154.9 day and the EB is probably the main component of the SB. This interesting system therefore contains both the longest period spectroscopic binary and the shortest period eclipsing binary known among the O-type stars. The system was precisely analyzed by van Leeuwen & van Genderen (1997). This triple system is one member of the visual binary FIN 313Aa, which has been measured 32 times since 1951. The orbit is not very conclusive, because the change in position angle is only about 15° . No orbit was derived.

4.7.16 HD 66094

HD 66094 (V635 Mon, A 1580AB, BD-08 2186) is an Algol-type EB with primary star classified as a spectral type F5 and apparent brightness of about 7.31 mag in V filter. A lot of times of minima were collected (altogether 113), but these data points follow the

linear ephemeris without any indication of the proposed LITE. On the other hand the astrometric orbit is defined very precisely. 23 data points measured over the century cover about a half of the orbit, and the analysis results in 160-yr orbit, with the semimajor axis of about 280 mas.

4.7.17 HD 71581

HD 71581 (VV Pyx, B 2179AB, HR 3335) is an Algol-type EB, spectrum A1V, brightness 6.58 mag in V and orbital period about 4.6 days. There were 7 times of minima found in literature (1976 - 1983), but these data show very long apsidal motion (in order of decades or centuries). The astrometric orbit is also covered only very poorly (11 observations obtained during 38 years show the change in position angle of about 13°).

4.7.18 HD 74956

HD 74956 (δ Vel, HR 3485, HIP 42913) is an Algol-type eclipsing binary classified as A1V spectral type, with $V = 1.95$ mag. The star was discovered to be a photometrically variable in 1997 (see Otero et al. (2000) for details), the period of such variation is about 45 days. Altogether 8 times of minima were collected, but these data indicates very long apsidal motion (on the timescale of centuries). Astrometric orbit consists of 37 measurements, which define the orbit with the period of about 142 years and the semimajor axis of about $2''$ (according to Alzner & Argyle 2000). The whole system is in fact more complicated, consists of two proper motion pairs ($2''$ and $6''$) separated by $69''$. Also the primary component was resolved as a double star interferometrically. We therefore deal with a quintuple system (at least 5 components).

4.7.19 AC UMa

AC UMa (ARG 21B, BD+65 671B, AG+65 453) is an Algol-type EB, spectrum classified as A2, brightness 10.3 in V filter. The orbital period is about 6.85 days. There were 69 times of minima found in literature and there could be some variation in order of decades, but this is only hypothesis, larger data set is needed. The astrometry is shown in Fig.

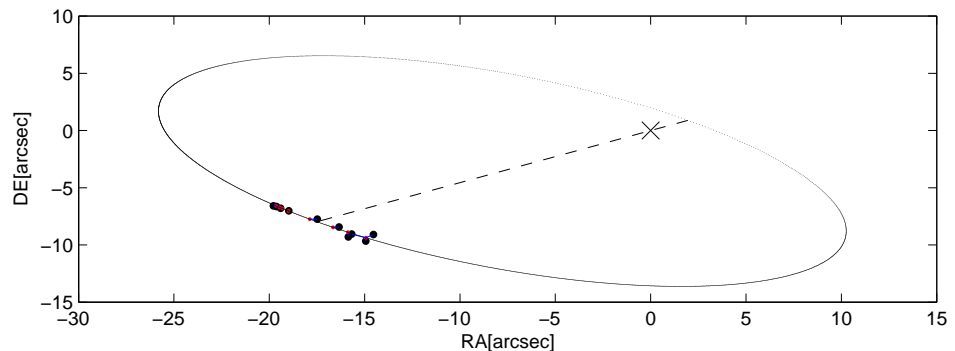


Figure 4.28: Relative orbit of AC UMa on the plane of the sky.

4.28, only 10 observations during 106 years were obtained. As one can see, only a linear part of the orbit is covered by data, so one cannot derive the parameters of the orbit precisely. This leads to extremely long period about 1200 yr, which could be even higher.

4.7.20 HD 82780

HD 82780 (DI Lyn, A Hya, STF 1369AB, HR 3811) is an Algol-type EB, classified as F2V. Its magnitude is $V = 6.76$ mag. There is only one time of minimum measured and the small arc of the astrometric orbit was observed during 22 years, covering about 30 degrees. No acceptable solution could be found.

4.7.21 HD 91636

HD 91636 (TX Leo, 49 Leo, STF 1450AB, HR 4148) is an Algol-type EB. Its spectrum was classified as A2V and its apparent brightness is about $V = 5.67$ mag. There were 6 times of minima observed since 1930. The astrometric data set is much larger, about 132 measurements secured during the last 180 years, but the motion is undetectable.

4.7.22 HD 101205

HD 101205 (V871 Cen, I 422AB, HIP 56769) is a β Lyrae type EB with its spectrum classified as O8V and the brightness of about $V = 6.49$ mag. There is a brief paper on the photometric observations of V871 Cen, together with a minimum time derived (see Mayer et al. 1992). The astrometry secured during the last 90 years reveals the change in θ about 20° , but any acceptable solution could be found (these data lead to the orbit of period about 4500 yr).

4.7.23 HD 101379j

HD 101379j (GT Mus, 12 Mus, B 1705AB) is an eclipsing binary, with spectrum classified as G2III and the brightness $V = 5.17$ mag. The astrometric data were obtained during 60 years and cover about 130° of the orbit. This SB1-type spectroscopic binary was analyzed by Parsons (2004). The orbital period of GT Mus is about 56 days, which is different from the period of the SB1 binary. Therefore, the component A is the EB, while B is the SB. No minima were derived. The astrometric orbit has a period circa 91 years (Parsons 2004). Some observations indicates that one of the components is RS CVn-type star.

4.7.24 HD 103483

HD 103483 (DN UMa, 65 UMa A, HR 4560) is an Algol-type EB, sp A3Vn, $V = 6.54$ mag, orbital period 1.73 days. There were only twelve times of minima found in published literature (the first ones from 1979). The astrometric orbit is covered sufficiently, the first astrometric observation came from 1908, and the parameters of the orbit are known ($p_3 = 136.5$ yr, $a = 230$ mas, according to Aristidi et al. 1999).

4.7.25 HD 110317j

HD 110317j (VV Crv, STF 1669AB, HIP 61910) is an eclipsing binary with the spectrum classified as F5IV and the brightness of about $V = 5.27$ mag. There were no times of minima found in the literature. The astrometric data set consists of 156 measurements secured during 180 years, which yielded the change in θ of about only 14° .

4.7.26 HD 114529

HD 114529 (V831 Cen, SEE 170AB, HR 4975) is the β Lyrae system, sp B8V, $V = 4.58$ mag, orbital period of about 0.64 d. No published minima were found. The astrometric orbit was derived according to 40 observations secured during the last 100 years, resulting in $p_3 = 27$ yr and $a = 185$ mas (according to Finsen 1964).

4.7.27 SAO 45318

SAO 45318 (ET Boo, COU 1760, HIP 73346) is a β Lyrae eclipsing binary, spectral type F8. Its apparent brightness is $V = 9.09$ mag. There were found a few times of minima, covering the last 5 years. There is possibly some variation in $O - C$ diagram, but its amplitude is only about 0.001 days and the period about 1.25 years. On the other hand the astrometric measurements were obtained since 1978 till 1999, altogether 20 observations show the change in θ about 40° . The orbit was derived by Seymour (2001), resulting in period about 113 yr and angular semimajor axis 261 mas.

4.7.28 HD 133640

HD 133640 (44i Boo, STF 1909AB, HR 5618) is a well-known EB of W UMa type, spectral type G0Vnv and brightness of about $V = 4.76$ mag. It is quite a complicated system, consists of more than three components. Many times of minima were observed during the last 90 years, but the detailed description of the behaviour of these minima is still missing (mass transfer + LITE ?). It was found to exhibit flares as well as to be an X-ray binary and also many analyses in this part of spectra were obtained. The large astrometric data set consists of 753 observations secured during the last 223 years, and covers the range of position angle from 240 down to 57 degrees. The orbit has period of about 206 yr and semimajor axis $3.8''$ (see Söderhjelm 1999).

4.7.29 HD 148121

HD 148121 (V1055 Sco, B 872AB, HIP 80603) is β Lyrae EB with spectral type classified as G3V and brightness $V = 8.64$ mag. There were no times of minima found in the literature. Astrometric measurements were obtained 12 times during the last 70 years covering about 15° in position angle. The orbit was not derived.

4.7.30 HD 157482

HD 157482 (V819 Her, MCA 47, HR 6469) is an Algol-type EB, spectrum analyzed as F9Vn and its magnitude is about 5.57 in V filter. The EB pair is orbiting around the common center of mass with the third component on the 5.5 years orbit with eccentricity 0.67, LITE is evident. This is the only system where the LITE was analyzed together with the other methods, namely the interferometry and RV (see Muterspaugh et al. 2006).

4.7.31 HD 163708

HD 163708 (V1647 Sgr, HIP 88069) is an Algol-type EB with the spectrum classified as A3III and the relative brightness $V = 6.8$ mag. A few dozens of times of minima are available, showing very slow apsidal motion (in order of centuries). The astrometric measurements were obtained 15 times during 170 years and covering about 14 degrees in position angle, see Fig.4.29. The orbit was computed first time and was not published yet, but the result is not very convincing due to poor coverage of the orbit by data points. The period is about 1200 yr and semiamplitude about $7.7''$.

4.7.32 HD 174932

HD 174932 (COU 510, BD+24 3555, SAO 86519) is a member of visual binary COU 510. The system also does not belong to this list, because the star was incorrectly classified as an eclipsing binary by Couteau (1972) and designated as JZ Her. HD 174932 itself is not a variable star it was mixed up with the close eccentric eclipsing variable HS Her. The wrong comment in the WDS notes will be soon corrected.

4.7.33 HD 178125

HD 178125 (18 Aql, Y Aql, HEI 568AB, HR 7248) is a spectroscopic variable, spectral type B8III, brightness about 5.07 mag in V filter. The astrometric orbit of Y Aql leads to the period of about 58 yr. In fact, the star does not belong to this survey, because it

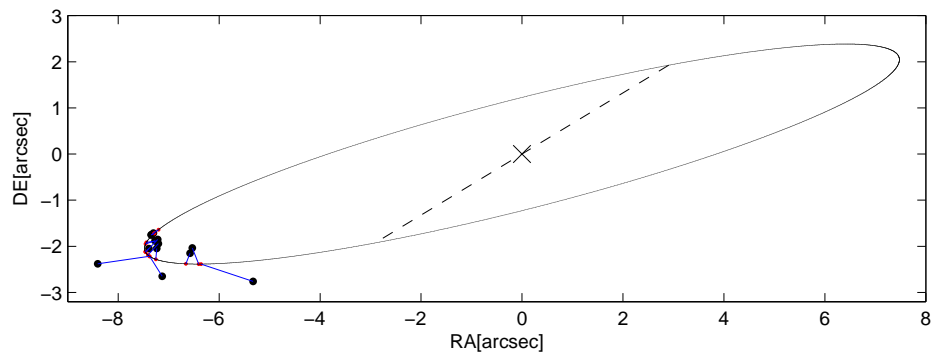


Figure 4.29: Relative orbit of V1647 Sgr on the plane of the sky.

is probably not eclipsing, but only the ellipsoidal one (recent observations and also data from Hipparcos indicate this possibility).

4.7.34 HD 184242

HD 184242 (V2083 Cyg, A 713AB, HIP 96011) is an Algol-type EB, spectral type A3, apparent brightness $V = 6.88$ mag and orbital period about 1.9 days. There were no times of minima found in published literature. The astrometry covers about 70° during the last century. The orbit was computed by Seymour et al. (2002), resulting in period about 372 yr and angular semimajor axis about 498 mas.

4.7.35 HD 195434

HD 195434 (MR Del, AG 257AB, HIP 101236) is an Algol-type EB, spectral type classified as K0 and $V = 11.01$ mag. There were found a few times of minima, covering 2451700-2452100 HJD and the astrometry from 1902 to 2001, with the change in position angle of about 15° . This leads to the orbit with period of about 6500 yr.

4.7.36 HD 201427

HD 201427 (BR Ind, HU 1626AB, HIP 104604) is an Algol-type EB, spectral type F8V and with its apparent brightness of about 7.1 mag in V filter. Astrometry was obtained since 1914 till 2001, when the position angle has changed from 208 down to 124 degrees. These data lead to the orbital parameters $p_3 = 167$ yr and $a = 894$ mas (according to Seymour et al. 2002). No times of minima were found in literature.

4.7.37 HD 217675

HD 217675 (α And, 1 And, BLA 12Aa+WRH 37AB, HR 8762) is the pulsating Be star, as well as a shell star. Its apparent brightness is $V = 3.63$ mag and spectrum was classified as B6IIIpe. The system is more complicated, consisting of at least 4 components (see e.g. Pavlovski et al. 1997), while the visual triple consists of Aa-B components. Both orbits (the longer one with period 68.6 yr, according to Hartkopf et al. (1996) and the shorter one with period 8.9 yr, according to Olević & Jovanovic 1999) were observed and derived. Some authors (see e.g. Schmidt 1959) published the light curves with the possible eclipsing behavior of the star, but nowadays it is rather improbable for the star to be an eclipsing binary. The photometric variability is probably due to the variability of the shell around the star.

The survey of 37 systems with eclipsing components in visual binaries is not a complete one. On the other hand it could be taken as a representative sample of the most interesting ones, because these systems have the largest data sets in astrometry and some of them have also the times-of-minima observations.

Scanning the WDS catalogue and trying to find the eclipsing binaries in this sample there were found a lot of *variable stars* (according to Simbad catalogue). From this sample of variable stars there could be a significant number of eclipsing binaries, but only future photometric observations would reveal the nature of this variability. Also a few systems mentioned above are classified in Simbad as *variable stars* or *ellipsoidal variables*, although they are EBs. Some of the binaries were also find to be wrongly identified.

Chapter 5

Discussion and conclusions

The method of period analysis of eclipsing binaries and its modifications were presented. The method of $O - C$ diagram analysis is not new, but the new aspects were also included into the code. The possibility that the third body resulting from the LITE analysis is also detectable via astrometry was discussed. With this assumption the modified algorithm of simultaneous solution of LITE and astrometry was presented.

The theoretical explanation of the effect and the method used is presented in chapter 2. It deals with the relative astrometry only and also the parallax of the system was assumed as a priori known (mainly from *Hipparcos* satellite). The distance is needed for the transformation between the angular and absolute semimajor axis in both methods. Also the principal limitations for both methods are presented in chapter 2. These limitations have to be considered, especially when one has only very poor data in one of the methods.

LITE systems: In the chapter 3.1 there was presented the application of the LITE analysis on eleven particular systems. These systems have never been studied for the presence of the third component and LITE hypothesis is able to describe their long-term minimum times behavior. On the other hand there were neither detailed spectroscopic, nor photometric analysis of these systems and the third body hypothesis presented here cannot be proven.

Although the number of systems, where the astrometric orbit together with LITE is known, is growing steadily, in the most cases only very limited coverage of the orbit, both in astrometry and times of minima is available. Especially due to this reason the combined analysis of these systems is still difficult. There were found only a few appropriate candidates for such an analysis. These cases were studied and discussed in detail and the principal limitations of the method were pointed out. On the other hand the method itself is very powerful and efficient. It could be even modified for the estimation of the distance to the suitable kind of binaries.

During the last decade a few papers combining the approach of simultaneous solution of radial velocities, spectral analysis, astrometry, *Hipparcos* measurements or LITE were published. Besides the systems mentioned in the introduction (44 Boo, QZ Car, SZ Cam, GT Mus) there were also the analysis of V1061 Cyg (combining the light curve analysis, radial velocity analysis, light-time effect and *Hipparcos* measurements, see Torres et al. 2006), the papers where the radial velocity measurements and astrometry were combined

(see Muterspaugh et al. (2006) for the solution of the LITE system V819 Her, or Gudehus (2001) for μ Cas), the paper on HIP 50796 combining the radial-velocity measurements with the *Hipparcos* abscissa data (see Torres 2006), or the paper on δ Lib comparing the results from the period analysis, light-curve analysis, spectral analysis, radio emission and astrometry, respectively; see Budding et al. (2005).

Such a combined analysis is very important, and the individual methods could be tested. Their independent results have to be in agreement with each other. The method presented here is also the combination of the two independent methods into the one joint solution. It was never been done before in this way. Various modifications were presented, but most similar was the analysis of the LITE together with the *Hipparcos* observations, and the absolute astrometry by Ribas et al. (2002).

The code itself is presented in section 2.9. It could be downloaded from the web sites and it is ready to be used. Short description of the code is presented and also the brief manual is available. Only slight modifications of the algorithm are necessary before the first run of the code. The numerics and the computing time required for the code strongly depends on the initial parameters and the input data (their quality and the size of the data set), but it could be slightly improved, see section 2.8.

A few eclipsing binaries were studied in this thesis. Detailed analysis was performed for QS Aql, VW Cep, ζ Phe, V505 Sgr, HT Vir and V2388 Oph. These systems have relatively best coverage both in astrometry and LITE variation. This is the crucial part, as one can see in the case of VW Cep, which is the most suitable system for the simultaneous analysis.

If precise measurements and good coverage of at least one period of the distant body in both methods are available, the presented method is very powerful and the parameters of the distant-body orbit could be derived very precisely. Even the distance of the system could be computed with high confidence level. The limiting factor is mainly the coverage of the orbit. In most of the cases the orbit was not covered sufficiently with data. Also almost all the systems included in the catalogue in chapter 4.7 have only poor coverage of the orbit by data in both methods.

VW Cep: The case where both methods have relatively best coverage of the orbit is VW Cep. In this case the resultant parameters of the third body satisfies the limit for the luminosity, and also the systemic velocity variations coincide with our hypothesis. New results are comparable with the previous ones. An additional fourth body was introduced to describe the long-term variation in times of minima, as well as in radial velocities. The system is probably more complicated than was assumed (chromospheric activity cycles, stellar spots and flares), and it was decided to explain only the most pronounced effects in the $O - C$ diagram. Using the combined approach it is possible to derive the parallax to VW Cep more precisely than in any other previous papers, resulting in $\pi = (35.85 \pm 0.37)$ mas. The two different approaches (LITE₃ + LITE₄ and LITE₃ + mass transfer) were used and their results compared. Both approaches lead to the approximately same results both in astrometry and times-of-minima analysis. The simultaneous analysis is able to describe the system in its complexity and one has to disagree with the result by Pribulla et al. (2000), that the astrometric orbit could not be identified with the LITE₃ variation from the $O - C$ diagram. As one can see, our new

results are in agreement with each other without any problems. On the other hand only further observations of this system will decide which approach (Solution I. or II.) is the right one. Two new times of minima were observed at Ondřejov Observatory.

QS Aql: In the case of QS Aql the parameters of the distant-body orbit were mainly derived from the LITE analysis, because the coverage of the astrometric orbit is very poor and the old data are not very reliable. Due to this difficulty, the inclination of the orbit could not be derived precisely. The computed value of the inclination is quite low and the error quite high. Low inclination dictates high mass of the third body (but with large errors). The new derived mass of the third body is in contradiction with the previous photometric analysis, but one can get the consistent result within the error of this value.

ζ Phe: The system ζ Phe displays an apsidal motion together with the LITE and this explanation fits the $O - C$ residuals quite well. This is the first time when the apsidal motion together with the LITE hypothesis were applied to this system. The astrometric analysis of ζ Phe is complicated due to the fact that the period of the third body orbit is circa 3 times longer than the interval covered by the data. The time span of the minima measurements is even worse, only about one fourth of the orbit is covered. On the other hand the powerful combined analysis was able to estimate all of the parameters of the third-body orbit precisely. This approach lead to the period of the third body about 220 years and the parameters of such a body, its predicted mass and the spectral type is in an excellent agreement with the previous photometric analysis.

V505 Sgr: V505 Sgr is the system, where both the astrometry and also recent times of minima observations deviate from the predicted trend. Despite the fact the third body was detected more than 20 years ago, it could be even observable in the spectrum of the system, the complex figure of the system is still missing. The new result from the combined approach is in contradiction with the previous results from photometry and also spectroscopy. It indicates that the third body observable in spectra and light curve is different from the fourth body observable astrometrically. Only further detailed analysis would prove this hypothesis.

HT Vir: The eclipsing system HT Vir is the case where the new value of mass of the distant body is about 2 times larger than one would expect. The distant component in the system is also a double and from the spectroscopy one is able to derive an upper limit for its mass. Regrettably, our new result is in contradiction with such a mass. This could be due to only a few times of minima observed in the linear part of the $O - C$ diagram, new minima are needed in the next decades. Four new times of minimum light were observed. On the other hand the astrometric orbit is well-defined and almost whole orbit is covered.

V2388 Oph: The last system is V2388 Oph, where two different approaches were used. The astrometric variation is rapid and since its discovery the third component has revolved a few times around the primary. On the other hand, the times of minima were obtained only rarely during the last decade. Due to this reason, the rapid change in order of 9 years is hardly detectable from the $O - C$ diagram analysis and one could speculate about the inclination of the orbit. If the orbit's inclination is close to 180° , there could be no LITE evident in the $O - C$ diagram, which was presented as another explanation and the mass transfer was suggested as an alternative explanation for the $O - C$ diagram. Only further times of minima would prove or refuse this hypothesis. Another possible explanation is

that the system is quadruple and the third body observable interferometrically is not the one which causes LITE.

The final result is that the method itself is potentially very powerful but it is also very sensitive to the quality of the input data, especially if the method is used for determining the distance of these binaries. It can only be applied successfully in those cases where the astrometric orbit and the LITE in the $O - C$ diagram are well defined by existing observations and lead to the approximately same parameters of the distant-body orbit. This is necessary condition, as one can see for example from the case V505 Sgr.

The catalogue: The catalogue of another suggested systems for the prospective simultaneous analysis with the introduced algorithm was presented in the chapter 4.7. The main purpose of the catalogue was to critically consider the potential objects for such a combined approach and from the eclipsing binaries in the spatially resolvable systems identify those, which are suitable for the introduced method.

During the inspection of such systems there were found a few binaries which were often presented as eclipsing binaries, but which are in fact not. These are for example 18 Aql (=Y Aql) which is an ellipsoidal variable, *o* And which is photometrically variable, but it is not due to the eclipses, or V640 Cas which is probably also not an eclipsing variable. Also one misidentification of the EB was presented (HD 174932).

On the other hand there were found a few systems which are the most suitable ones for the method presented here. Such systems are for example V348 And, V592 Per, V635 Mon, DN UMa, V831 Cen, ET Boo, or 44i Boo.

An additional material is also available via the web pages¹. The code for computing the combined analysis could be downloaded together with the brief manual and instructions for the user. On the same web pages there are also the complete data files, which were used as the input files for the analysis.

¹[http : //sirrah.troja.mff.cuni.cz/ ~ zasche/](http://sirrah.troja.mff.cuni.cz/~zasche/)

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